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船舶全局一致渐进路径跟踪变积分增益导航策略

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摘要: 本文利用一种新的导航策略和反馈控制器解决了欠驱动船舶受到外部扰动作用下的保持直线运动问题. 与传统的视野线(line-of-sight, LOS)导航策略相比, 改进的LOS导航策略具有变积分增益能够补偿外部环境扰动引起的侧滑效应并且能够避免积分饱和影响, 其中积分增益是以垂直距离误差为函数引导船舶灵活快速地趋向期望的轨迹. 本文所提出的积分导航策略和基于积分器反演控制策略组成一种串联结构的系统, 并且证明了当所有控制目标实现时整个系统是全局一致渐进稳定的. 仿真结果说明了所提出内容的有效性和性能.

关键词: 路径跟踪; 导航策略; 视野线(LOS); 反演法; 全局一致渐进稳定

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Uniformly globally asymptotically stable path following with integral gain-variable guidance law for ships

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Abstract: This paper addresses the problem of straight-line path following for underactuated ships exposed to constant external force by utilizing a novel guidance law and dynamic feedback controller. Compared to the conventional line-of-sight (LOS), The modified LOS guidance law with variable integral gain is proposed to compensate for sideslip influence so as to copy with the environmental force and avoid integral windup as well, in which arguments and integral gain computed as a function of cross-track error. This guidance law can conduce the ships to converge to desired path in a elegant and faster manner. The proposed integral guidance law and the control strategy based on integrator backstepping technique make up a cascaded structure which is proved to be uniformly globally asymptotically stable (UGAS) when the target tasks are all achieved. Simulation results have demonstrated the effectiveness and capability of the proposed control and guidance scheme.

Key words: path following; guidance law; line-of-sight(LOS); backstepping; uniformly globally asymptotically stable (UGAS)

1 Introduction

As we know, in order to develop and utilize the marine natural resources and reduce transportation cost at sea, the researches of marine control system have gained extensive attentions. The ability to maneuver a ship to track or follow a given path is of primary importance in guidance and control system which are basic methodologies concerned with the achievement of marine motion control objectives.

In guidance system, the LOS guidance law is a three-point guidance scheme since it involves a stationary reference point on path in addition to the ship and the desired positions, what's more, it acts like a helmsman who commonly make the ship purse the desired path through steering it towards a point lying a constant distance ahead of the ship on the path, which is called the look-ahead distance, therefore, the three points are derived in the meantime. In maritime applications, path following control techniques based on the LOS guidance law has been broadly applied because of its nice properties[1]. The conventional line-of-sight guidance was described in the book [2], although it has been employed for missiles successfully, it is needed to modify the guidance law before it's suitable to maritime applications at the existence of disturbances because of its drawback of being susceptible to environmental disturbance. An modified LOS guidance law with integral action was proposed to handle environmental disturbances such as constant ocean, winds and waves[3], but the property of reducing integrator wind-up wasn't enough intuitive and obvious, and the fixed integral gain may amplify

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the integral action unnecessarily. Afterwards, the integral LOS guidance law paired with nonlinear controller for marine surface vessels with saturated transverse actuators was presented by Caharija and Pettersen[4], so as to compensate the drift caused by sideslipping in the presence of constant ocean currents.

There are few proofs of LOS guidance law with compensation of sideslip angle before[5], however, the effect of drift angle was omitted and its proposed synthesized controller guaranteed the global k-exponential stability of cross-tracking error in the literature [6]. The backstepping design methodology, described in [2], is a recursive design methodology and strongly related to feedback linearization. However, when designing nonlinear backstepping controllers the designers can exploit good nonlinearities in the system which the feedback linearization cancel, and bad or destabilizing nonlinearities are dominated by adding nonlinear damping[2]. In the article [7], the coordinated path following control problem for a group of underactuated ships are addressed by using the combination of Lyapunov direct method and backstepping, this path following controller forces each underactuated ship to follow a predefined path subject to external constant disturbances[8]. In [9], a control law based on nonlinear adaptive backstepping was proposed to copy with unknown system parameters and environmental disturbances, but the path can be kept within a tracking error globally. Wave impact was also taken into account to improve the performance of path following in the wave fields by introducing a numerical test bed[10].

The geometric error was explicitly employed in the design procedure and convergence to the desired path was guaranteed by an alternative controller with modified speed assignment according to it, which is the cross-track error between marine surface vehicle and the path[11], unfortunately, the dynamic task was sacrificed when the craft moved off the path. Shortly afterwards, they applied a least-square approach to fully marine craft and adjusted the speed likewise to ensure cross-track error in the presence of ocean currents[12].

However, traditional LOS guidance is computationally simple, intuitive and easy to tune, and easily influenced by external forces such as ocean currents, wind and waves which will affect the motion of vehicles and give rise to lateral acceleration, consequently, the occurrence of sideslip angle due to lateral acceleration can destabilize the system. In addition, the drawback cannot be avoided by simply adding integral action since the source of the problem root in LOS steering law itself. It has been confirmed that the LOS guidance law needed to be modified by including a term related to the drift angle to stabilize the cross-tracking error around the desired equilibrium point[5]. It is worthy clarifying that sideslip angle is equal to zero when the sway velocity is zero or the ship move forward without external disturbance, and the total velocity is equal to surge velocity. But during a turn, the sideslip angle is nonzero due to the total velocity is separated into two parts–sway velocity and surge velocity. And on the other side, it becomes necessary to counteract the influence from external disturbance especially constant disturbance by compensating sideslip angle in order to follow straight line. At this point, it is clearly important to underline that the compensation of drift angle need to be done no matter whether the vehicles are suffered from external force.

In this paper, motivated by the research [5], we propose to compensate for sideslip angle directly at first, though the procedure of the proof takes full advantage of the trigonometric function and is less computationally expensive. Then two guidance laws combined with nonlinear integrator backstepping controller, which compensate the drift angle directly and indirectly, are designed to steer the ship follow desired path. And they not only contribute to avoid integrator saturation and compensate for the influence of disturbances both directly and indirectly, but keep the integrated system uniformly globally asymptotically stable.

1.1 System model and problem statement

In this section, we present the control objective and the mathematical model of ship considered in this paper.

1.2 Ship model

It is commonly sufficient to consider a 3 DOF horizontal nonlinear maneuvering model in the form[1]:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu}, \tag{1}$$

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{N}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} + \boldsymbol{R}^{\mathrm{T}}(\psi)\boldsymbol{b}, \tag{2}$$

where $\eta = [x \quad y \quad \psi]^{\mathrm{T}} \in \mathbb{R}^2 \times [-\pi, \pi]$ represents earth-fixed position coordinates and heading angle, $\boldsymbol{\nu} = [u \quad v \quad r]^{\mathrm{T}} \in \mathbb{R}^3$ represents the body-fixed velocities, and

$$\boldsymbol{R}(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \in SO(3) \tag{3}$$

is the rotation matrix. The inertia matrix $\boldsymbol{M}$ and the matrix $\boldsymbol{N}$ are defined as

$$\boldsymbol{M}=\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix},\ \boldsymbol{N}(\boldsymbol{\nu})=\begin{bmatrix} n_{11} & 0 & 0 \\ 0 & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix},$$

we assume that the matrices in maneuvering model (1)–(2) satisfy the properties: $\boldsymbol{M}=\boldsymbol{M}^{\mathrm{T}}$ and terms in $\boldsymbol{N}(\boldsymbol{\nu})$ is positive. The symmetry property of inertia matrix $\boldsymbol{M}$ is needed in Lyapunov stability analysis of backstepping design procedure. $\boldsymbol{\tau}=[\tau_{\mathrm{u}},\tau_{\mathrm{v}},\tau_{\mathrm{r}}]$ is the body-fixed propulsion forces and moments, and all the low-frequency environmental forces and moments acting on ship, which are caused by wind, ocean current and second order wave loads, is collected in the earth-fixed vector $\boldsymbol{b}$. By the following rotation, the transformation from earth-fixed coordinates $\boldsymbol{b}$ into body-fixed coordinates is achieved:

$$\boldsymbol{\tau}=\boldsymbol{R}(\boldsymbol{\psi})^{\mathrm{T}}\boldsymbol{b}. \tag{4}$$

Since these disturbances are bounded and so slowly varying compared to the ship dynamics, we can assume that $\dot{\boldsymbol{b}}=0$ and $\boldsymbol{b}\in\mathcal{L}_\infty$ in the controller synthesis.

1.3 Problem statement

The primary objective of this paper is to design a control and guidance system to steer the ship to converge to the desired straight-line path $\mathcal{P}$ with desired speed u_{d} which can be starting from any positions with some orientations. Convergence to the path, which is referred to as the geometric task, is formulated as

$$\lim_{t\to\infty} d(t)=0, \tag{5}$$

where $d(t)$ is cross-tracking error.

It is required that the ship converge to the path as smoothly as desired. Thus, the heading angle has to track desired angle, that is

$$\lim_{t\to\infty}(\psi-\psi_{\mathrm{d}})=0, \tag{6}$$

the desired heading angle ψ_{d} will determined later.

2 Guidance and control system design

2.1 Line-of-sight guidance system

Figure 1 indicates geometry of the line-of-sight guidance principle and involves main variables. In this paper, the LOS guidance system based on look-ahead steering method is employed. Consider a straight-line $\mathcal{P}$ as the desired path, the slope of the path is defined as $\psi_k\in[-\pi,\pi]$. At each of time, the LOS vector starts from the ship's position $\boldsymbol{p}(x,y)$ and end to the point $\boldsymbol{p}_{\mathrm{los}}(x_{\mathrm{los}},y_{\mathrm{los}})$ which is located on the straight path at a lookahead distance $\varDelta>0$ ahead of the direct projection of $\boldsymbol{p}$ onto the path. The orientation of the LOS vector is donated $\boldsymbol{\psi}_{\mathrm{LOS}}$, i.e., the LOS angle, as Fig. 1 shows. In LOS guidance system, let the moving point $\boldsymbol{p}_{\mathrm{los}}$ be the desired point that the ship moves towards at each time instant. It means that the heading angle ψ must be aligned along the angle of LOS vector at each of instant time. Thus, the corresponding LOS guidance law is given by

$$\psi_{\mathrm{LOS}}=\psi_k+\arctan(-\frac{d}{\varDelta}), \tag{7}$$

where ψ_k is the slope of the path, and ψ_{LOS} is the desired course angle of the ship. At the same time, aligning the heading angle ψ along the angle ψ_{LOS} may result in a nonzero cross-tracking error when the total speed of ship is not aligned with the x-axis of the body-fixed reference frame. Thus , a better alternative is to align the total speed with desired course angle ψ_{LOS} instead of aligning the x-axis of the body-fixed reference frame, that is to say, to compensate for sideslip angle β. The compensation essentially implies that the desired heading angle is computed using the following equation:

$$\psi_{\mathrm{d}}=\psi_{\mathrm{LOS}}-\beta, \tag{8}$$

where $\beta=\mathrm{atan}(v/u)$.

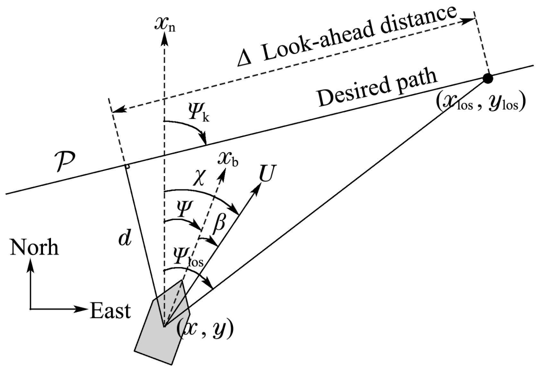

Fig. 1 The illustration of LOS guidance law

Locate a point $\boldsymbol{p}_k$ at the straight path, and denote the distance between $\boldsymbol{p}$ and $\boldsymbol{p}_{\mathrm{los}}$ by $\boldsymbol{\sigma}$: $\boldsymbol{\sigma}\triangleq[s\ \ d]^{\mathrm{T}}$ where $d(t)$ is the cross-tracking error and $s(t)$ is the along-tracking error. Thus, it can be derived by the transformation of reference frame:

$$\boldsymbol{\sigma}=R^{\mathrm{T}}(\psi_k)(\boldsymbol{p}-\boldsymbol{p}_k). \tag{9}$$

Differentiate the equation (9), we can obtain $\dot{\boldsymbol{\sigma}}=R^{\mathrm{T}}(\psi-\psi_k)\boldsymbol{\nu}$, where $\boldsymbol{\nu}=[u\ \ v]^{\mathrm{T}}$. Consequently, the cross-track error is

$$\begin{aligned}\dot{d}=&\ u\sin(\psi-\psi_k)+v\sin(\psi-\psi_k)=\\ &\sqrt{(u^2+v^2)}\sin(\psi-\psi_k+\beta)=\\ &U\sin(\psi-\psi_k+\beta).\end{aligned} \tag{10}$$

For the analysis of stability, the equation (10) is rewritten to be

$$\begin{aligned}\dot{d}=&\ U\sin(\psi_{\mathrm{d}}-\psi_k+\beta)+U[\sin(\psi-\psi_k+\beta)-\\ &\sin(\psi_{\mathrm{d}}-\psi_k+\beta)]\end{aligned} \tag{11}$$

with the help of the basic transformation of trigonometric function and the heading angle error $z_0 = \psi - \psi_{\rm d}$, equation(11) can be farther derived as

$$\dot{d} = -\frac{Ud}{\sqrt{\Delta^2 + d^2}} + 2U\sin(\frac{1}{2}z_0)\cos(\psi_k - \psi_{\rm d} - \beta - \frac{1}{2}z_0). \quad (12)$$

By substituting equations (7)−(8), we get

$$\dot{d} = -\frac{Ud}{\sqrt{\Delta^2 + d^2}} + 2U\sin(\frac{1}{2}z_0)\cos(\arctan\frac{d}{\Delta} - \frac{1}{2}z_0) \quad (13)$$

and the equation (11) can be reformulated to be

$$\dot{d} = f_{\rm d}(d,t) + g_{\rm d}(d,\xi), \quad (14)$$

where $f_{\rm d}(d,t) = U\sin(\psi_{\rm d} - \psi_k + \beta)$ and $g_{\rm d}(d,\xi(z_0)) = 2U\sin(\frac{1}{2}z_0)\cos(\arctan\frac{d}{\Delta} - \frac{1}{2}z_0)$, $\xi = \xi(z_0)$ is a vector about d and will be determined in Section 4.

Remark 1 The systems (1)−(2) and the guidance systems (7)−(8) are interconnected and constitute the nonlinear cascade system. Consider Eqs.(13)−(14), it can be seen that not only the desired heading angle $\psi_{\rm d}$ influences the guidance system, but also the heading error $z_0 = \psi - \psi_{\rm d}$ does. Furthermore, the function $g_{\rm d}(d,\xi)$ gives how the heading error dynamic affects the cross-tracking error and make the system track $U\sin(\psi_{\rm d} - \psi_k + \beta)$ as softly and smoothly as possible.

However, even though the various means of direct measurement of β are considered (see [5]), these methods become much more difficult to be handled when external slowly-varying forces such as the current acts on the vehicle. Adding the integral action is an alternative way to compensate the sideslip angle effects for cancelling environmental disturbances. Compared with the traditional LOS guidance, the integral gain-variable LOS (ILOS) guidance is designed to enable the underactuated vehicles to follow the straight-line paths under the influence of environmental disturbances such as current, wind or waves by adding integral action into the former. To this end, the following LOS guidance law with variable integral gain is proposed:

$$\psi_{\rm mILOS} \triangleq -\arctan(K_{\rm p}d + {\rm e}^{-\rho|d|}y_{\rm d}),$$

$$\dot{y}_{\rm d} = \frac{{\rm e}^{-\rho|d|}y_{\rm d}}{\sqrt{1 + (K_{\rm p}d + {\rm e}^{-\rho|d|}y_{\rm d})}}, \quad (15)$$

$$\psi_{\rm d} = \psi_k + \psi_{mLOS}, \quad (16)$$

where $\rho > 0$ is design parameter and $K_{\rm p} = 1/\Delta > 0$. The idea behind (15) is that the integral of cross term d is only used to keep $\psi_{\rm mILOS}$ nonzero when a state-state off-track condition is detected (i.e. $d \neq 0$), and when the vehicle follows the desired paths (i.e. $d = 0$) in the situation where environmental disturbances drive the vehicle away from the given path. In this ILOS guidance law (15), the variable integrating gain will make the integral term less dominant when the vehicle is far from the desired paths to avoid overshoot and saturation effects, in other words, ${\rm e}^{-\rho|d|} \to 1$ as $d \to \infty$. Thus, the persistent accumulation of the nonzero integral term will generate the side-slip angle.

3 Control design method

When designing marine control systems it is clearly important to add integral action to control law to avoid steady-state error and compensate for slowly-varying disturbance. In this section, a model-based control method is performed by utilizing the integrator backstepping for the nonlinear maneuvering system (1)–(2), and this design is divided into two coherent steps.

The overall design procedures are moving on based on the these following conditions: all the reference signals needed in design, the desired heading $\psi_{\rm d}$ and its higher order derivatives $\dot{\psi}_{\rm d}$ and $\ddot{\psi}_{\rm d}$, and the desired surge speed $u_{\rm d}$ and its derivative $\dot{u}_{\rm d}$, are assumed to be bounded in signal space $\mathcal{L}_\infty$, i.e., $\psi_{\rm d}$, $\dot{\psi}_{\rm d}$, $\ddot{\psi}_{\rm d}$, $u_{\rm d}$, $\dot{u}_{\rm d} \in \mathcal{L}_\infty$, and this assumption is reasonable in practice.

The change of coordinates, that is, the error variables $z_0 \in \mathbb{R}$ and $\boldsymbol{z} \in \mathbb{R}^3$ is defined as

$$z_0 = \psi - \psi_{\rm d} = \boldsymbol{h}^{\rm T}\boldsymbol{\eta} - \psi_{\rm d}, \quad (17)$$

$$\boldsymbol{z} \triangleq [z_1 \ \ z_2 \ \ z_3]^{\rm T} = \boldsymbol{\nu} - \boldsymbol{\alpha}, \quad (18)$$

where $\boldsymbol{\alpha} \triangleq [\alpha_1 \ \ \alpha_2 \ \ \alpha_3] \in \mathbb{R}^3$ is a vector of stabilizing functions to be specified later, and let $\boldsymbol{h} \in \mathbb{R}^3$ be the projection vector $\boldsymbol{h} \triangleq [0 \ \ 0 \ \ 1]^{\rm T}$.

Step 1 Let the control objective to be the convergence of the error signal z_0 to zero. To this effect, define the first control Lyapunov function (CLF) as

$$V_1 \triangleq \frac{1}{2}z_0^2 \geqslant 0. \quad (19)$$

By differentiating the equation(17) as a function of time, it can be derived that

$$\dot{z_0} = \dot{\psi} - \dot{\psi}_{\rm d} = \boldsymbol{h}^{\rm T}\boldsymbol{\nu} - \dot{\psi}_{\rm d} = \boldsymbol{h}^{\rm T}(\boldsymbol{z} + \boldsymbol{\alpha}) - \dot{\psi}_{\rm d}, \quad (20)$$

since $\dot{\eta} = \boldsymbol{R\nu}$. Differentiating (19) along the trajectory of z_0-dynamics yields

$$\dot{V}_1 = z_0\dot{z_0} = z_0\boldsymbol{h}^{\rm T}\boldsymbol{z} + z_0(\alpha_3 - \dot{\psi}_{\rm d}). \quad (21)$$

Considering the purpose of this step, the stabilizing function $\alpha_3(z_0)$ is linearized about z_0, that is to say, the stabilizing function $\alpha_3(z_0)$ is chosen to be

$$\alpha_3(z_0) = -k_0z_0 - \dot{\psi}_{\rm d}, \quad (22)$$

where $k_0 > 0$, which acts like a feedback control as well and results in

$$\dot{V}_1 = -k_0 z_0^2 + z_0 \boldsymbol{h}^{\mathrm{T}} \boldsymbol{z}. \tag{23}$$

This concludes Step 1.

Step 2 The derivative of $\boldsymbol{M}\boldsymbol{z}$ is now expressed as

$$\boldsymbol{M}\dot{\boldsymbol{z}} = \boldsymbol{M}(\dot{\boldsymbol{\nu}} - \dot{\boldsymbol{\alpha}}) = \boldsymbol{\tau} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{b} - \boldsymbol{N}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{M}\dot{\boldsymbol{\alpha}}. \tag{24}$$

At this point we need to augment the first CLF and design $\boldsymbol{\tau}$ render its derivative nonpositive, we consider this augmented CLF:

$$V_2 = V_1 + \frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{z} + \frac{1}{2}\tilde{\boldsymbol{b}}^{\mathrm{T}}\Gamma^{-1}\tilde{\boldsymbol{b}}, \tag{25}$$

where $\tilde{\boldsymbol{b}} \in \mathbb{R}^3$ is adaptation error defined as: $\tilde{\boldsymbol{b}} = \hat{\boldsymbol{b}} - \boldsymbol{b}$ and $\Gamma = \Gamma^{\mathrm{T}} > 0$ is the adaptation gain matrix. The $\hat{\boldsymbol{b}}$ is the estimate of the environmental disturbance vector $\boldsymbol{b}$, and it should be noted that the assumption $\dot{\boldsymbol{b}} = \boldsymbol{0}$ means that $\dot{\tilde{\boldsymbol{b}}} = \dot{\hat{\boldsymbol{b}}}$. Derivative equation (25) along the trajectories of $z_0, \boldsymbol{z}, \tilde{\boldsymbol{b}}$, using equations (23)$-$(25), we get

$$\begin{aligned}\dot{V}_2 = &-k_0 z_0^2 + z_0\boldsymbol{h}^{\mathrm{T}}\boldsymbol{z} + \boldsymbol{z}^{\mathrm{T}}\boldsymbol{M}\dot{\boldsymbol{z}} + \tilde{\boldsymbol{b}}^{\mathrm{T}}\Gamma^{-1}\dot{\tilde{\boldsymbol{b}}} = \\ &-k_0 z_0^2 + \boldsymbol{z}^{\mathrm{T}}(\boldsymbol{h}z_0 + \boldsymbol{\tau} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{b} - \boldsymbol{N}\boldsymbol{\nu} - \\ &\boldsymbol{M}\dot{\boldsymbol{\alpha}}) + \tilde{\boldsymbol{b}}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\boldsymbol{b}}}.\end{aligned} \tag{26}$$

To make clearness, $\boldsymbol{N}$ is used instead of $\boldsymbol{N}(\boldsymbol{\nu})$. Substituting the $\boldsymbol{\nu} = \boldsymbol{z} + \boldsymbol{\alpha}$ and $\boldsymbol{b} = \hat{\boldsymbol{b}} - \tilde{\boldsymbol{b}}$ into (25) yields

$$\begin{aligned}\dot{V}_2 = &-k_0 z_0^2 + \boldsymbol{z}^{\mathrm{T}}(\boldsymbol{\tau} + \boldsymbol{R}^{\mathrm{T}}\boldsymbol{b} - \boldsymbol{N}(\boldsymbol{z} + \boldsymbol{\alpha}) - \\ &\boldsymbol{M}\dot{\boldsymbol{\alpha}} + \boldsymbol{h}z_0) + \tilde{\boldsymbol{b}}^{\mathrm{T}}\Gamma^{-1}(\dot{\hat{\boldsymbol{b}}} - \Gamma\boldsymbol{R}\boldsymbol{z}).\end{aligned} \tag{27}$$

Thus, the parameter adaptation law

$$\dot{\hat{\boldsymbol{b}}} = \Gamma\boldsymbol{R}\boldsymbol{z} \tag{28}$$

makes the third term of (27) zero. By assigning the control input to be

$$\boldsymbol{\tau} = \boldsymbol{N}\boldsymbol{\alpha} - \boldsymbol{K}\boldsymbol{z} - \boldsymbol{R}^{\mathrm{T}}\hat{\boldsymbol{b}} + \boldsymbol{M}\dot{\boldsymbol{\alpha}} - \boldsymbol{h}z_0, \tag{29}$$

where $\boldsymbol{K} = \mathrm{diag}\{k_1, k_2, k_3\} > 0$ is a positive definite design matrix, finally we obtain negative definite $\dot{V}_2$ by

$$\dot{V}_2 = -k_0 z_0^2 - \boldsymbol{z}^{\mathrm{T}}(\boldsymbol{N} + \boldsymbol{K})\boldsymbol{z} < 0,\ \forall z_0 \neq 0,\ \boldsymbol{z} \neq \boldsymbol{0} \tag{30}$$

and according to standard Lyapunov arguments, this result guarantees the boundedness of $(z_0, \boldsymbol{z})$ and their convergence to zero. As for sway-unactuated ships of which the common actuator confirmation is a main propeller and a rudder, however, there is no redundant actuator to deliver sway force independently, as a result, the force of τ_{v} couldn't be assigned directly. In general, the sway force τ_{v} is set to be zero instead references [1, 13]. Whereas note that the coupled relationship between sway and yaw from equations (1)$-$(2), one can reasonably infer that the rudder deflection may result in sway force and then influences the sway dynamics. Consequently, we can obtain

$$\tau_{\mathrm{v}} = l_\tau \tau_{\mathrm{r}}, \tag{31}$$

where l_τ is the moment arm from the controlled point, midship or center of gravity (CG), to the rudder placed aft. Now that α_2 is still unknown and the sway dynamics α_2 is not controlled directly, one can attempt to find an update law for α_2 to satisfy that the computed force τ_{v} in equation (29) conforms to the physical constraints of the system, that is, $\tau_{\mathrm{v}} = l_\tau \tau_{\mathrm{r}}$. Consider the expressions of the second and third element of vector $\boldsymbol{\tau}$, and reformulate these resultant equations, the following dynamic equation is derived:

$$\bar{m}_2\dot{\alpha}_2 = -\bar{n}_2\alpha_2 + \gamma(\alpha_3, \dot{\alpha}_3, \hat{\boldsymbol{b}}, z_0, \boldsymbol{z}), \tag{32}$$

where

$$\begin{aligned}&\bar{m}_2 = m_{22} - l_\tau m_{32} > 0,\ \bar{n}_2 = n_{22} - l_\tau n_{32} > 0,\\ &\gamma(\alpha_3, \dot{\alpha}_3, \hat{\boldsymbol{b}}, z_0, \boldsymbol{z}) = \\ &-l_\tau z_0 + (l_\tau n_{33} - n_{23})\alpha_3 + (l_\tau m_{33} - m_{23})\dot{\alpha}_3 + \\ &(\hat{b}_1\cos\psi - \hat{b}_2\sin\psi - l_\tau\hat{b}_3) + k_2 z_2 - l_\tau k_3 z_3.\end{aligned}$$

Theorem 1 The origin of the error system $(z_0, \boldsymbol{z}, \tilde{\boldsymbol{b}})$ for the 3 degree of freedom (DOF) underactuated ship model (1)–(2) is uniformly globally asymptotically stable (UGAS) and uniform local exponential stability (ULES) by using the control law (29) and disturbance adaptation law (28) and choosing the stabilizing function $\boldsymbol{\alpha}$ as $\alpha_1 = u_{\mathrm{d}}$, numerical integration of (32) and (22) respectively.

Remark 2 The smooth reference signal $\psi_{\mathrm{d}}, \dot{\psi}_{\mathrm{d}}$, and $\ddot{\psi}_{\mathrm{d}}$ are provided by ILOS guidance system, while $u_{\mathrm{d}} \in \mathcal{L}_\infty$ is given by operator and its derivative $\dot{u}_{\mathrm{d}}$ must be bounded.

Proof Firstly, based on the equations (20)(24)(28), we establish the explicit error dynamics for the $(z_0, \boldsymbol{z}, \tilde{\boldsymbol{b}})$-system. Define the subsystems:

$$\Sigma_1:\ \dot{\Theta} = \boldsymbol{h}(\Theta, t) + B(t)\tilde{\boldsymbol{b}}, \tag{33}$$

$$\Sigma_2:\ \dot{\tilde{\boldsymbol{b}}} = -\Gamma\boldsymbol{B}^{\mathrm{T}}\boldsymbol{M}_{\mathrm{error}}^{\mathrm{T}}\Theta, \tag{34}$$

where

$$\begin{aligned}&\Theta \triangleq [z_0\ \ z_1\ \ z_2\ \ z_3]^{\mathrm{T}},\ \boldsymbol{h}(\Theta, t) = -\boldsymbol{M}_{\mathrm{error}}^{-1}\boldsymbol{N}_{\mathrm{error}},\\ &B(t) = -\boldsymbol{M}_{\mathrm{error}}^{-1}\boldsymbol{R}_{\mathrm{error}}^{\mathrm{T}},\\ &\boldsymbol{M}_{\mathrm{error}} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & m_{11} & 0 & 0\\ 0 & 0 & m_{22} & m_{23}\\ 0 & 0 & m_{32} & m_{33}\end{bmatrix},\end{aligned}$$

$$\boldsymbol{R}_{\text{error}} = \begin{bmatrix} 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{N}_{\text{error}} = \begin{bmatrix} k_0 & 0 & 0 & 0 \\ 0 & n_{11}+k_1 & 0 & 0 \\ 0 & 0 & n_{22}+k_2 & n_{23} \\ 0 & 0 & n_{32} & n_{33}+k_3 \end{bmatrix}.$$

The subsystems (33) and (34) constitute the whole time-varying nonlinear $(z_0, \boldsymbol{z}, \tilde{\boldsymbol{b}})$-system. For the obtained system (33) is nonautonomous, it is not suitable for Krasovskii-Lasalle's theorem. An alternative theorem for this case was stated in [14]. For the remainder of the proof, readers can refer to the proof in [14].

In conclusions, $\lim\limits_{t\to\infty} z_0 = \lim\limits_{t\to\infty}(\psi - \psi_{\mathrm{d}}) = 0$ and $\lim\limits_{t\to\infty} z_1 = \lim\limits_{t\to\infty}(u - u_{\mathrm{d}}) = 0$ by choosing $\alpha_2 = u_{\mathrm{d}}$, then the tasks of tracking the desired heading angle and desired surge velocity are all fulfilled at the same time. In view of the theorem1 and its criteria, it can be infer that the driving term γ of α_2-subsystem is bounded, i.e. $\gamma(\alpha_3, \dot{\alpha}_3, \hat{\boldsymbol{b}}, z_0, \boldsymbol{z}) \in \mathcal{L}_\infty$. This implies that the α_2-subsystem is input-to-state stable (ISS) from γ to α_2 since $\bar{m}_2 > 0$ and $\bar{n}_2 > 0$. Moreover, the unforced α_2-subsystem ($\gamma = 0$) is clearly exponentially stable. It is also straight-forward to show that $\alpha_2 \in \mathcal{L}_\infty$, $z_2 \in \mathcal{L}_\infty \Rightarrow$ the sway speed $v \in \mathcal{L}_\infty$, it means that the unactuated sway dynamics is globally bounded.

4 The interconnection behaviour between guidance and control systems

Again here consider the $\dot{d}$-subsystem (14) and error dynamic $(z_0, \boldsymbol{z}, \tilde{\boldsymbol{b}})$-system, we establish the new error state vectors: $\varPhi \triangleq [d \quad \xi]^{\mathrm{T}} \in \mathbb{R}^8$, and $\xi = [z_0 \quad \boldsymbol{z} \quad \tilde{\boldsymbol{b}}]^{\mathrm{T}}$. The $\dot{d}$-subsystem (14) and the $(z_0, \boldsymbol{z}, \tilde{\boldsymbol{b}})$-system equations can be rewritten respectively as

$$\varSigma_3:\ \dot{d} = f_{\mathrm{d}}(d,t) + g_{\mathrm{d}}(d,\xi), \tag{35}$$

$$\varSigma_4:\ \dot{\xi} = f_{\xi}(\xi,t). \tag{36}$$

The above systems constitute the cascaded structure which embraces the control and guidance systems, as is depicted in Fig. 2.

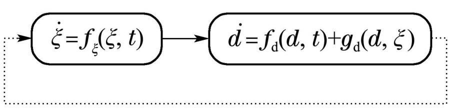

Fig. 2 The brief illustration of the cascaded system

Hence we go on to state the following theorem.

Theorem 2 The origin $\varPhi = \boldsymbol{0}$ of cascaded systems $\varSigma_3$ and $\varSigma_4$ is uniformly globally asymptotically stable if the desired heading angle ψ_{d} is provided by guidance law (7) and the drift angle is compensated as well according to equation (8).

Proof Note that the the cascade interconnection, the proof may be carried out by the applying the subsystems' stability for the control and guidance system. In this paper, we will borrow the lemma 2.1 in [15] to prove uniform global asymptotic stability. We start by the subsystem $\varSigma_4$ (the perturbing system): it should be stressed that the direct conclusion of the theorem1 states that the origin $\xi = \boldsymbol{0}$ of the systems $\varSigma_4$ is UGAS by utilizing the control law (29)and adaptation law (28). Then, to present one of several sufficient conditions for UGAS, it is needed to consider the *unforced*-system of the system $\varSigma_3$: $\dot{d} = f_{\mathrm{d}}(d,t)$ and the Lyapunov function candidate (LFC):

$V_{\text{unforced,d}} = (1/2)d^2$, so the time derivative of LFC along d is

$$\dot{V}_{\text{unforced,d}} = -\frac{U}{\sqrt{\varDelta^2 + d^2}} d^2 \leqslant 0.$$

Clearly, this shows that the *unforced*-system is UGAS and ULES.

Because the surge speed u and sway speed v are all bounded, the total speed $U = \sqrt{u^2+v^2} \in \mathcal{L}_\infty$, in other words, $0 < U < U_{\max}$. Regarding the $g_{\mathrm{d}}(d,\xi)$, it also belongs to $\mathcal{L}_\infty$ for $g_{\mathrm{d}}(d,\xi) \leqslant 2U_{\max}$. As a consequence of all the driving terms' boundedness, the solution of $d(t)$ is uniformly globally bounded (UGB). On the other hand, the solutions of the subsystem $\varSigma_4 : \dot{\xi} = f_{\xi}(\xi,t)$ are clearly UGB on the basis of the content front. These bounded solutions result in the solutions of $\varPhi$ are also UGB. And now, the cascaded structure can be concluded to be UGAS by exploiting the Lemma, see [15] for the cascade:

$$\mathrm{UGAS}_{\xi} + \mathrm{UGAS}_{\text{unforced,d}} + \mathrm{UGB}_{\varPhi} \Leftrightarrow \mathrm{UGAS}_{\varPhi}.$$

At this point, the cascade systems $\varSigma_3$ and $\varSigma_4$ are demonstrated that it has a uniformly globally asymptotically stable equilibrium at $\varPhi = \boldsymbol{0}$.

Theorem 3 When the underactuated ship (1)–(2) is exposed to constant environmental force, convergence to the desired path is achieved uniformly globally asymptotically if the desired heading angle is given by (15).

Proof we can rewrite (10) as follows:

$$\dot{d} = U\sin(-\arctan(K_{\mathrm{p}}d + \mathrm{e}^{-\rho|d|})) = -\frac{(K_{\mathrm{p}}d + \mathrm{e}^{-\rho|d|}d)}{\sqrt{1+(K_{\mathrm{p}}d + \mathrm{e}^{-\rho|d|}d)^2}}U. \tag{37}$$

Next, the fourth LFC is proposed:

$$V_{\mathrm{mILOS}} = \frac{1}{2}y_{\mathrm{d}}^2 + \frac{1}{2}d^2. \tag{38}$$

So the time derivative is computed as

$$\dot{V}_{\mathrm{mILOS}} = y_{\mathrm{d}}\dot{y}_{\mathrm{d}} + d\dot{d} = y_{\mathrm{d}}\dot{y}_{\mathrm{d}} + \frac{-(K_{\mathrm{p}} + \mathrm{e}^{-\rho|d|})Ud^2}{\sqrt{1+(K_{\mathrm{p}}d + \mathrm{e}^{-\rho|d|}d)^2}}. \tag{39}$$

Substituting $\dot{y}_{\mathrm{d}}$ in (15) yields

$$\dot{V}_{\mathrm{mILOS}} = \frac{-d^2}{\sqrt{1+(K_{\mathrm{p}}d + \mathrm{e}^{-\rho|d|}d)^2}}U \leqslant 0. \tag{40}$$

From (40), it can be concluded that the system (38) has a UGAS/ULES equilibrium point at $y_{\mathrm{d}} = 0$ with the adaptive integral gain-variable LOS guidance law (15).

5 Simulation

The proposed ILOS guidance law (15) and the control law (29), adaptation law (28) are simulated in MATLAB/Simulink, trying to force a underactuated ship model to follow a straight line while exposed to constant environmental disturbance. The controller used

$$\boldsymbol{M} = \begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix}, \ \boldsymbol{N} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0.1 \\ 0 & 0.1 & 0.5 \end{bmatrix}.$$

Accordingly, $\rho = 2$, $\varDelta = 3.5$, $k_0 = 10$, $k_1 = 10$, $k_2 = 5$, $k_3 = 10$, while $\varGamma = \boldsymbol{I}$. The initial ship pose and velocity are set to be $\eta_0 = [0\,\mathrm{m}\ \ 0\,\mathrm{m}\ \ \pi/2\,\mathrm{rad}]^{\mathrm{T}}$ and $\boldsymbol{\nu}_0 = [1\,\mathrm{m/s}\ \ 0\,\mathrm{m/s}\ \ 0\,\mathrm{rad/s}]$. And the straight-line path is chosen as the straight path with slope $\psi_k = \pi/3\,\mathrm{rad}$ which passes through the point$(5\,\mathrm{m}, 9\,\mathrm{m})$. The environmental forces can be bounded constant or slow variable, without loss of generality, here we assume that slow variable $\boldsymbol{b}$ oscillates around $[-5\,\mathrm{N}\ \ 5\,\mathrm{N}\ \ 0\,\mathrm{N}]$ on a small scale.

In Figs. 3−4, we make a comparison between the LOS guidance law (8) and the integral gain-variable LOS guidance law (15) with the same controller (29) to show the effectiveness of the LOS guidance law (15). And further more, compared to the law (8), which is based on actual measurement, in Fig. 4 it can be seen that the proposed ILOS guidance paired with controller drivers the underactuated ship onto predefined path with smaller error and more graceful action. See Fig. 6 and compared it with Fig. 5, as expected, the adaptive backstepping method with proposed LOS guidance law (15) results in the heading angle converging to a specified angle which is different from the slope of the given straight-line path, keeping the vehicle on the desired path, which means that it is no longer needed to measure the drift angle by applying the proposed ILOS guidance law.

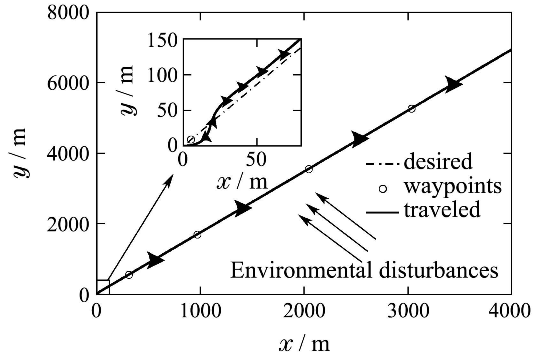

Fig. 3 The practical path generated by guidance law (8) based on the proposed controller

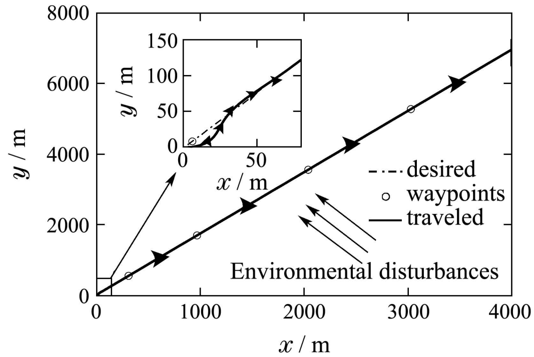

Fig. 4 The practical path generated by guidance law (15) based on the proposed controller

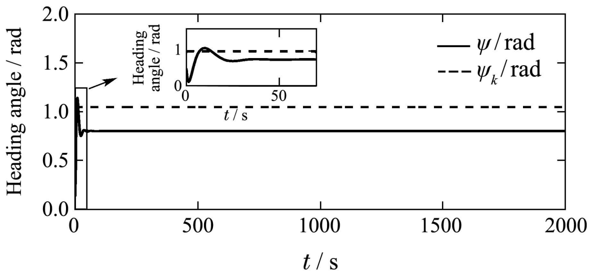

Fig. 5 The practical heading angle with guidance law (8)

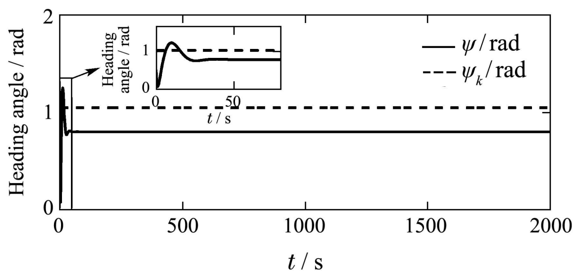

Fig. 6 The practical heading angle with guidance law (15)

6 Conclusions

This paper has addressed two relational problems pertaining to underactuated ships' control strategy and guidance law. Since the vehicle is exposed to constant external force, the two alternative compensation approaches of sideslip angle are considered to maneuver the vehicle to follow the desired path when steady-state off-track circumstance exists and the adaption law is added to controller at the same time. Moreover, UGAS is proven for the tracking error states. In particular, we have derived a adaptive integral gain-variable LOS guidance law capable of resisting the constant external force. Simulation results have illustrated the effectiveness and capability of the proposed control and guidance scheme.

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