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通信时延和联合连通拓扑下多刚体系统分布式姿态一致性控制

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摘要: 本文研究了通信时延和联合连通切换拓扑条件下的多刚体系统分布式姿态一致性控制问题. 通过构建有效的辅助向量并选择合适的Lyapunov-Krasovskii函数, 分别对恒定通信时延和时变通信时延两种不同情况下的控制器进行了设计. 数值仿真结果表明, 本文提出的方法能够有效地解决这类分布式姿态一致性控制问题.

关键词: 多刚体; 姿态一致性; 联合连通拓扑; 时变时延; Lyapunov函数

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Distributed attitude consensus for multiple rigid body systems with communication delay and jointly connected topologies

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Abstract: In this paper, we study the distributed attitude consensus problem for multiple networked rigid body systems in the case of communication time-delays together with jointly connected switching topologies. By constructing useful auxiliary vectors and choosing proper common Lyapunov-Krasovskii functions, we design two control laws for two different cases respectively, i.e., the case with constant communication time-delays, and the case with time varying communication time-delays. Numerical simulation shows that the proposed algorithms are effective to this kind of distributed attitude consensus problem.

Key words: multiple rigid bodies; attitude consensus; jointly connected topologies; time-varying delay; Lyapunov function

1 Introduction

Over the past decade, there has been significant research effort dedicated to attitude consensus control for a group of rigid body systems. One of the most important reasons to introduce multiple rigid bodies is that a group of smaller and less-expensive rigid bodies working together can achieve the same goal as a single large and expensive rigid body. For example, in interferometry applications, it is often essential to control different spacecraft to maintain the same or relative attitudes during and after formation manoeuvres. Since the angular velocity of the body can not be linearly integrated to obtain the attitude of the body directly[1] because of the nonlinear dynamics, attitude consensus control becomes a particularly interesting problem. The synchronized multiple spacecraft rotations control problem is solved with a passivity-based damping method in [2], while the condition that the angular velocity is unknown is considered in [3]. The other interesting problems include attitude consensus with time delay[4], with input constraints[5] and in case of multiple leaders[6]. Certainly, it will be more challenging if only a subset of group agents have access to the virtual leader[7].

Since the biggest difference between multi-agent systems and single-agent system lies in the communication network, the characteristics of networks decide the performance of the whole system to a great extent. Therefore, two interesting topics on attitude consensus problem have been extensively studied. One is on the communication time-delay, while the other is on the switching topologies.

In practical applications, time delays inevitably exist in the system and communication links, which may degrade the control performance of the formation and even destabilize the entire system. Lyapunov-like method is effective for time delay problems, i.e., constructing Lyapunov-Krasovskii function, Lyapunov-Razumikhin function and set-valued Lyapunov function. Adaptive attitude synchronization problem of spacecraft formation is studied in [8] with possible time delay. By introducing a novel adaptive control architecture, the authors develop effective decentralized controllers applying to the case with parameter uncertainties and unknown external disturbances. However, the

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attitude dynamics are expressed by Euler-Lagrange formula. In [9], the attitude synchronization problems are discussed for leaderless multiple spacecraft in the presence of constant communication delays. In [10], attitude coordination control of spacecraft formation is addressed in case that a constant reference attitude is available to only a part of the spacecraft, where multiple communication delays between spacecraft is considered. However, all these literatures only consider the case with constant time-delay.

In particular, the authors in [11] propose a virtual systems-based approach that removes the requirement of the angular velocity measurements, based upon which the leaderless and leader-follower problems with time-varying communication delays and undirected communication topology, and the leaderless problem under directed topology and constant communication delays are both solved. Then, this approach is extended to solve the cooperative attitude tracking control problem in [4]. Differently, the authors in [12] deal with the cooperative attitude tracking problem with time-varying delays as well as the delays between inter-synchronization control parts and self-tracking control parts. Moreover, in [13], a cooperative attitude control scheme is developed with model uncertainties, external disturbances and variable time delays, and the novelty lies in the strategy to construct such a Lyapunov function scarifying the L_2-gain dissipative inequation that ensures not only the stability of a cooperative attitude tracking formation system but also an L_2-gain constraint on the tracking performance. Distinguished from the existing literature where the delayed relative attitude is described via linear algorithm, the authors in [14] develop a new control law with the nonlinear nature of the employed quaternion based attitude coupling. However, most of the literature only consider the problem with undirected topology. In [15] and [16], the more complicated cases are considered even with unknown mass moment of inertia matrix, bounded external disturbances, actuator failures, and control saturation limits. Note that all aforementioned literatures assume that the communication topology is fixed.

Communication outage, new member's joining or quit-ting, radio silence or recovery will cause the change of the communication topology, named switching topologies, which makes it more difficult to design the control laws. Based on relative attitude information and modified Rodriguez parameters, cooperative attitude tracking problem is considered in [17] and a control law is given in the presence of a dynamic communication topology. This is extended in [18] to the condition that there exist both multiple time-varying communication delays and dynamically changing topologies. Considering more complicating elements, the authors in [19] present controllers that can render a spacecraft formation consistent to a given trajectory globally with dynamic information exchange graph and non-uniform time-varying delays while coping with the parameter uncertainties and unexpected disturbances. In [20], a 6-DOF dynamics model of the spacecraft formation flying is established in Euler-Lagrange form in the presence of dynamic communication topology. Furthermore, almost-global attitude synchronization is achieved in [21] based on switching and uniform connection, however auxiliary variables are introduced which make the controllers complicated. In [22], by utilizing Lyapunov direct method and choosing a common Lyapunov function properly, the robustness of the designed position and attitude coordinated controllers to communication delays, switching topologies, parameter uncertainties and external disturbances is guaranteed. However, all the above literatures only consider the uniformly connected switching topologies. It is worth mentioning that, the attitude synchronization problem of multiple rigid body agents in SO(3) is addressed in [23] with directed and jointly strongly connected interconnection topologies. And from the viewpoint of interior metrics, the authors in [24] provide a leaderless consensus protocol for strongly convex geodesic balls and applies it to the consensus problem of rotation attitudes under switching and directed communication topologies. Note that, these two do not consider communication time delay.

Different from [8–16] only considering time delays, [20–24] only considering dynamic topologies, and [18], [19] considering time delays coupled with uniformly connected switching topologies, we focus on the attitude consensus problem in case of communication time-delay together with jointly connected switching topologies. The difficulty lies in how to design the proper control law, which is not only used to the nonlinear attitude dynamics, but also used to the attitude consensus problem under jointly connected switching topologies coupled with varying time-delay. By constructing a proper auxiliary vector, together with the common Lyapunov-Krasovskii function method, we design different protocols for the connected agents and the isolated agents respectively, and effectively solve the consensus problem.

The remainder of this paper is organized as follows. In Section 2, we present the dynamics of rigid body attitude, basic knowledge of graph theory, especially jointly connected switching graphs, and the statement of attitude consensus problem. The details about the construction of auxiliary vectors as well as derivation of the controllers are presented in Section 3. In Section 4, we show simulation results for four rigid bodies using the control laws proposed in Section 3 and conclusion follows in Section 5.

Notation $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geqslant 0}$:

$= [0, \infty)$. $\|\boldsymbol{A}\|_2$ is the spectral norm of matrix $\boldsymbol{A}$. $|\boldsymbol{x}|$ stands for the standard Euclidean norm of the vector $\boldsymbol{x} \in \mathbb{R}^n$. For any function $\boldsymbol{f}: \mathbb{R}_{\geqslant 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\boldsymbol{f}\|_\infty = \sup t \geqslant 0\, |\boldsymbol{f}(t)|$, and the $\mathcal{L}_2$-norm as $\|\boldsymbol{f}\|_2^2 = \int_0^\infty |\boldsymbol{f}(t)|^2\, \mathrm{d}t$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\boldsymbol{f}: \mathbb{R}_{\geqslant 0} \to \mathbb{R}^n : \|\boldsymbol{f}\|_\infty < \infty\}$ and $\{\boldsymbol{f}: \mathbb{R}_{\geqslant 0} \to \mathbb{R}^n : \|\boldsymbol{f}\|_2 < \infty\}$ respectively.

2 Problem statement and background information

2.1 Dynamics of multiple rigid bodies attitude

We consider a multiple rigid body system with n agents, which are labeled as agent 1 to n. The attitude dynamics of the ith agent is described by

$$\begin{aligned} \dot{\sigma}_i &= G(\sigma_i)\omega_i, \\ \dot{\omega}_i &= J_i^{-1}(-\omega_i^\times J_i\omega_i + u_i), \end{aligned} \tag{1}$$

where $\sigma_i \in \mathbb{R}^3$ denotes the modified Rodrigues parameters (MRPs) representing the attitude of the ith agent, $\omega_i = [\omega_{i1}\ \omega_{i2}\ \omega_{i3}]^\mathrm{T} \in \mathbb{R}^3$ denotes the angular velocity of the ith agent, J_i and u_i are the inertial matrix and the external input torque of the ith agent respectively. $\omega_i^\times$ is the skew-symmetric matrix with the form

$$\omega_i^\times = \begin{bmatrix} 0 & -\omega_{i3} & \omega_{i2} \\ \omega_{i3} & 0 & -\omega_{i1} \\ -\omega_{i2} & \omega_{i1} & 0 \end{bmatrix}. \tag{2}$$

The matrix $G(\sigma_i)$ is given by

$$G(\sigma_i) = \frac{1}{2}[\frac{(1-\sigma_i^\mathrm{T}\sigma_i)I_3}{2} + \sigma_i^\times + \sigma_i\sigma_i^\mathrm{T}], \tag{3}$$

which has the following properties[25–26]

$$\sigma_i^\mathrm{T} G(\sigma_i)\omega_i = \frac{1+\sigma_i^\mathrm{T}\sigma_i}{4}\sigma_i^\mathrm{T}\omega_i, \tag{4}$$

$$G(\sigma_i)G^\mathrm{T}(\sigma_i) = (\frac{1+\sigma_i^\mathrm{T}\sigma_i}{4})^2 I_3 = p_i I_3. \tag{5}$$

Remark 1 We hasten to point out that the use of MRPs simplifies the analysis and formulas proving processing, since there is no additional equality constraint to worry about. Another advantage is that MRPs can parameterize eigenaxis rotations up to 360°. In contrast, other three-dimensional parameterizations are limited to eigenaxis rotations of less than 180°. Refer to references [27–28] for more details.

Remark 2 The stability results presented in this paper mean the stability of the corresponding kinematic parameters. That is, the stability is guaranteed for all initial attitudes except for the singular point $\Phi_i = \pm 360°$, where Φ_i is the principle angle of the attitude of the ith rigid body.

2.2 Graph theory

Graphs can be conveniently used to represent the information flow between agents. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be an undirected graph or directed graph (digraph) of order n with the set of nodes $\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \cdots, v_n\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = \{a_{ij}\}$ with non-negative adjacency elements a_{ij}. The node indices belong to a finite index set $l = \{1, 2, \cdots, n\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_i, v_j)$, which is said to be incoming with respect to v_j and outgoing with respect to v_i. For an undirected graph, $\forall i, j \in l$, if $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, then $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$, but this does not hold for a digraph. A directed path from node i to node j is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \cdots$, in a directed graph. A digraph $\mathcal{G}_\mathrm{s} = \{\mathcal{V}_\mathrm{s}, \mathcal{E}_\mathrm{s}\}$ is a subgraph of $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ if $\mathcal{V}_\mathrm{s} \subseteq \mathcal{V}$ and $\mathcal{E}_\mathrm{s} \subseteq \mathcal{E} \cap (\mathcal{V}_\mathrm{s} \times \mathcal{V}_\mathrm{s})$. Given a set of r digraphs $\{\mathcal{G}_i = (\mathcal{V}, \mathcal{E}_i), i = 1, \cdots, r\}$, the digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{E} = \bigcup_{i=1}^r \mathcal{E}_i$ is called the union of digraphs $\mathcal{G}_i$, denoted by $\mathcal{G} = \bigcup_{i=1}^r \mathcal{G}_i$, and this is also used to the undirected graphs. The set of neighbors of node v_i is the set of all nodes which point (communicate) to v_i, denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \varepsilon(\mathcal{G})\}$. The graph adjacency matrix $\mathcal{A} = [a_{ij}], \mathcal{A} \in \mathbb{R}^{n\times n}$, is such that $a_{ij} > 0$ if $j \in \mathbb{N}_i$ and $a_{ij} = 0$ otherwise. $\mathcal{D} = \mathrm{diag}\{d_1, \cdots, d_n\} \in \mathbb{R}^{n\times n}$ is called the degree matrix of $\mathcal{G}$, where $d_i = \sum_{j=1}^n a_{ij}$. The weighted Laplacian matrix of $\mathcal{G}$ is $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Given a piecewise constant switching signal $\varrho(t)$, we can define a time-varying digraph $\mathcal{G}_{\varrho(t)} = (\mathcal{V}, \mathcal{E}_{\varrho(t)})$ where $\mathcal{E}_{\varrho(t)} \subseteq \mathcal{V} \times \mathcal{V}$ for all $t \geqslant 0$. $\mathcal{G}_{\varrho(t)}$ can be called a dynamic digraph. On the other hand, given a switching matrix $\mathcal{A}_{\varrho(t)} = [a_{ij}(t)] \in \mathbb{R}^{n\times n}$ satisfying, for any $t \geqslant 0$, $a_{ii}(t) = 0, i = 1, \cdots, n$, and $a_{ij}(t) \geqslant 0, i, j = 1, \cdots, n$, we can always define a dynamic digraph $\mathcal{G}_{\varrho(t)}$ so that $\mathcal{A}_{\varrho(t)}$ is the weighted adjacency matrix of the digraph $\mathcal{G}_{\varrho(t)}$. We call $\mathcal{G}_{\varrho(t)}$ the digraph of $\mathcal{A}_{\varrho(t)}$.

To model the jointly-connected topologies, we consider an infinite sequence of continuous, bounded, non-overlapping time intervals $[t_k, t_{k+1})$, $k = 0, 1, 2, \cdots$ with $t_0 = 0$, $T_0 \leqslant t_{k+1} - t_k \leqslant T_\mathrm{p}$ for some constants T_0 and T_p. Assume that each interval $[t_k, t_{k+1})$ is composed of the following non-overlapping subintervals $[t_k^0, t_k^1), \cdots, [t_k^{j-1}, t_k^j), \cdots, [t_k^{m_k-1}, t_k^{m_k})$ with $t_k^0 = t_k$ and $t_k^{m_k} = t_{k+1}$ for some nonnegative integer m_k. The topology switches at time instants $t_k^0, t_k^1, \cdots, t_k^{m_k-1}$, which satisfy $t_k^j - t_k^{j-1} \geqslant \tau, j = 1, \cdots, m_k$, with dwell time τ a positive constant, so that during each subinterval $[t_k^{j-1}, t_k^j)$, the interconnection topology $\mathcal{G}_{\varrho(t)}$ does not change. Note that in each interval $[t_k, t_{k+1})$, $\mathcal{G}_{\varrho(t)}$ is permitted to be disconnected. The graphs are said to be jointly connected across the time interval $[t, t+T_\mathrm{p}]$ with $T_\mathrm{p} > 0$ if the union of graphs $\mathcal{G}_{\varrho(t)} : s \in [t, t+T_\mathrm{p}]$ is connected[29].

Assumption 1 The communication topologies are undirected, and the collection of graphs in each interval $[t_k, t_{k+1})$ is jointly connected.

2.3 Problem statement

Consider the attitude dynamics of the multiple rigid body systems given by (1), the attitude consensus is reached if

$$\lim_{t\to\infty}|\sigma_i(t)-\sigma_j(t)|\to 0,\quad \lim_{t\to\infty}|\omega_i(t)-\omega_j(t)|\to 0,\ i,\ j\in l. \tag{6}$$

Before presenting our main results, we give the following useful lemmas at first.

Lemma 1[30] Define variables $e(t)=x_{\rm d}(t)-x(t)$, $\dot{x}_{\rm r}(t)=\dot{x}_{\rm d}(t)+\mathit{\Gamma}e(t)$, $r(t)=\dot{x}_{\rm r}(t)-\dot{x}(t)=\dot{e}(t)+\mathit{\Gamma}e(t)$, where $x_{\rm d}(t),x(t)\in\mathbb{R}^m$, $\mathit{\Gamma}\in\mathbb{R}^{m\times m}$ is a positive definite matrix. Let $e(t)=h(t)*r(t)$, where $*$ denotes the convolution product and $h(t)=L^{-1}(H(s))$ with $H(s)$ being an $m\times m$ strictly proper, exponentially stable transfer function, L^{-1} denotes the inverse transfermation of the Laplace manipulator. Then, $r(t)\in\boldsymbol{L}_2$ implies that $e(t)\in\boldsymbol{L}_2\cap\boldsymbol{L}_\infty$, $\dot{e}(t)\in\boldsymbol{L}_2$, e is continuous and $|e(t)|\to 0$ as $t\to\infty$. Besides, if $|r(t)|\to 0$ as $t\to\infty$, then $|\dot{e}(t)|\to 0$.

Lemma 2[31] Let $t_i(i=0,1,2,\cdots)$ be a sequence satisfying $t_0=0$, $t_{i+1}-t_i\geqslant\tau>0$. Suppose that a scalar continuous $V(t):[0,+\infty)\to\mathbb{R}$ satisfies

1) $V(t)$ is lower bounded;

2) $\dot{V}(t)$ is differentiable and non-positive on each interval $[t_i,t_{i+1})$;

3) $\ddot{V}(t)$ is bounded over $[0,+\infty)$ in the sense that there exists a positive constant ξ such that

$$\sup_{t_i\leqslant t\leqslant t_{i+1},i=0,1,2,\cdots}|\ddot{V}(t)|\leqslant\xi. \tag{7}$$

Then $\dot{V}(t)\to 0$ as $t\to+\infty$.

3 Main results

In this section, we deal with the distributed attitude consensus problem with time-delay and jointly connected switching topologies in two cases, i.e., constant time-delay coupled with jointly connected topologies and varying time-delay coupled with jointly connected topologies. By constructing proper auxiliary vectors, we establish different effective control protocols to solve both the two problems.

3.1 Attitude consensus with constant time-delay and switching topologies

We associate each agent with the following auxiliary signal

$$e_i=\dot{\sigma}_i+c\sigma_i, \tag{8}$$

where c is a positive constant.

Note that for jointly connected topologies, there may exist isolated agents during some time intervals. Therefore, we divide the whole system into two subsets, i.e., the connected agent set $l_{\rm c}$ and the isolated agent set $l_{\rm s}$, where $l_{\rm c}\bigcup l_{\rm s}=l$ and $l_{\rm c}\bigcap l_{\rm s}=\varnothing$. The control law is designed as (9), where $j\in\mathbb{N}_i$, $a_{ij}(t)$ is the i,j entry of the weighted adjacency matrix $\mathcal{A}_{\varrho(t)}$,

$$\eta_i=1/\sum_{j=1}^n a_{ij}(t),\ \eta_j=1/\sum_{i=1}^n a_{ji}(t),$$

$\gamma>0$ is a constant,

$$e_i(t-T)=\dot{\sigma}_i(t-T)+c\sigma_i(t-T),$$

and $T>0$ is the constant time-delay.

$$u_i=\begin{cases}\omega_i^\times J_i\omega_i-\dfrac{J_iG_i^{\rm T}}{p_i}[\dot{G}_i\omega_i+c\dot{\sigma}_i+\gamma\sum\limits_{j=1}^n a_{ij}(t)(\dfrac{\eta_i+\eta_j}{2}e_i-\eta_je_j(t-T))], & i\in l_{\rm c},\\ \omega_i^\times J_i\omega_i-\dfrac{J_iG_i^{\rm T}}{p_i}[\dot{G}_i\omega_i+c\dot{\sigma}_i+\gamma(e_i-e_i(t-T))], & i\in l_{\rm s}.\end{cases} \tag{9}$$

Combining (1)(5)(8) and (9), the derivative of e_i can be written as

$$\dot{e}_i=\begin{cases}-\gamma\sum\limits_{j=1}^n a_{ij}(t)(\dfrac{\eta_i+\eta_j}{2}e_i-\eta_je_j(t-T)), & i\in l_{\rm c},\\ -\gamma\left[e_i-e_i(t-T)\right], & i\in l_{\rm s}.\end{cases} \tag{10}$$

The results are as follows:

Theorem 1 Under Assumption 1, with the control input (9), the attitude consensus of system (1) is achieved as (6).

Proof Define a Lyapunov-Krasovskii function as $V(t)=\sum\limits_{j=1}^n V_i(t)$ for the system, where

$$V_i(t)=\frac{\gamma}{2}\int_{t-T}^t e_i^{\rm T}(\tau)e_i(\tau){\rm d}\tau+\frac{1}{2}e_i^{\rm T}e_i. \tag{11}$$

It is worth noting that $V(t)$ is continuously differentiable in spite of the existence of the switching topologies.

According to the control input (9), we divide $V(t)$ into two parts, i.e., $V(t)=V_{\rm c}(t)+V_{\rm s}(t)$, where $V_{\rm c}(t)=\sum\limits_{i\in l_{\rm c}}V_i(t)$, $V_{\rm s}(t)=\sum\limits_{i\in l_{\rm s}}V_i(t)$.

The derivative of $V_{\rm c}(t)$ is given by

$$\begin{aligned}&\dot{V}_{\rm c}(t)=\\&\frac{\gamma}{2}\sum_{i\in l_{\rm c}}[e_i^{\rm T}e_i-e_i^{\rm T}(t-T)e_i(t-T)]+\sum_{i\in l_{\rm c}}e_i^{\rm T}\dot{e}_i=\\&\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\eta_i\sum_{j=1}^n a_{ij}(t)[e_i^{\rm T}e_i-e_i^{\rm T}(t-T)e_i(t-T)]-\\&\gamma\sum_{i\in l_{\rm c}}e_i^{\rm T}\sum_{j=1}^n a_{ij}(t)[\frac{\eta_i+\eta_j}{2}e_i-\eta_je_j(t-T)]=\\&-\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\sum_{j=1}^n a_{ij}(t)\eta_je_i^{\rm T}e_i=\\&+\gamma\sum_{i\in l_{\rm c}}\sum_{j=1}^n a_{ij}(t)\eta_je_i^{\rm T}e_j(t-T)-\end{aligned}$$

$$\frac{\gamma}{2}\sum_{i\in l_c}\sum_{j=1}^{n}a_{ij}(t)\eta_i e_i^{\mathrm{T}}(t-T)e_i(t-T) =$$
$$-\frac{\gamma}{2}\sum_{i\in l_c}\sum_{j=1}^{n}a_{ij}(t)\eta_j[e_i-e_j(t-T)]^{\mathrm{T}}\times$$
$$[e_i-e_j(t-T)]\,, \tag{12}$$

and the derivative of $V_s(t)$ is given by

$$\dot{V}_s(t) =$$
$$\frac{\gamma}{2}\sum_{i\in l_s}\left[e_i^{\mathrm{T}}e_i - e_i^{\mathrm{T}}(t-T)e_i(t-T)\right] + \sum_{i\in l_s}e_i^{\mathrm{T}}\dot{e}_i =$$
$$\frac{\gamma}{2}\sum_{i\in l_s}\left[e_i^{\mathrm{T}}e_i - e_i^{\mathrm{T}}(t-T)e_i(t-T)\right] -$$
$$\gamma\sum_{i\in l_s}e_i^{\mathrm{T}}\left[e_i - e_i(t-T)\right] =$$
$$-\frac{\gamma}{2}\sum_{i\in l_s}\left[e_i^{\mathrm{T}}e_i + e_i^2(t-T) - 2e_i^{\mathrm{T}}e_i(t-T)\right] =$$
$$-\frac{\gamma}{2}\sum_{i\in l_s}[e_i - e_i(t-T)]^{\mathrm{T}}\left[e_i - e_i(t-T)\right], \tag{13}$$

where we have used the fact that

$$\sum_{i\in l_c}\sum_{j=1}^{n}a_{ij}(t)\eta_i e_i^{\mathrm{T}}(t-T)e_i(t-T) =$$
$$\sum_{i\in l_c}\sum_{j=1}^{n}a_{ij}(t)\eta_j e_j^{\mathrm{T}}(t-T)e_j(t-T),$$

which is based on the assumption that $\mathcal{G}_{\varrho(t)}$ is undirected.

It is obvious that $V(t) \geqslant 0$ and $\dot{V}(t) \leqslant 0$, then it follows that $\lim\limits_{t\to\infty}V(t) = V(+\infty)$ exists, thus $e_i \in \mathcal{L}_\infty$ and $e_i - e_j(t-T) \in \mathcal{L}_2$. Note that $e_i = \dot{\sigma}_i + c\sigma_i$, the Laplace transfer function from e_i to σ_i is $H(s) = 1/(s+c)$. As $c > 0$, the transfer function $H(s)$ is stable. Because $e_i \in \mathcal{L}_\infty$, we get that $\sigma_i(t) \in \mathcal{L}_\infty$, and $\dot{\sigma}_i(t) \in \mathcal{L}_\infty$, thus $\dot{e}_i(t) \in \mathcal{L}_\infty$ according to (10). Taking the derivatives of $\dot{V}_c(t)$ and $\dot{V}_s(t)$, we can get that

$$\ddot{V}_c(t) = -\gamma\sum_{i\in l_c}\sum_{j=1}^{n}a_{ij}(t)\eta_j[e_i-e_j(t-T)]^{\mathrm{T}}\times$$
$$[\dot{e}_i - \dot{e}_j(t-T)]\,, \tag{14}$$
$$\ddot{V}_s(t) = -\gamma\sum_{i\in l_s}[e_i - e_i(t-T)]^{\mathrm{T}}\left[\dot{e}_i - \dot{e}_i(t-T)\right]. \tag{15}$$

Obviously, the boundedness of $\ddot{V}(t)$ is dependent on the boundedness of $e_i(t)$, $e_i(t-T)$, $\dot{e}_i(t)$ and $\dot{e}_i(t-T)$. Based on the above analysis, we conclude that all these signals are bounded, then $\ddot{V}(t)$ is bounded. By invoking Lemma 2, we get $\lim\limits_{t\to\infty}\dot{V}(t) = 0$.

Next, we will show that the control goal (6) can be reached. From Assumption 1, there exists $M > 0$ such that $\forall k \geqslant M$, in particular we choose k such that the time interval $[t_k, t_{k+1})$ encompasses some time intervals across which the agents are jointly connected. For the connected agents, the connectivity of the network across the time interval guarantees that $\lim\limits_{t\to\infty} e_i - e_j(t-T) = 0$. Define $\sigma_{ij} = \sigma_i - \sigma_j(t-T)$, we get that $e_i - e_j(t-T) = \dot{\sigma}_{ij} + c\sigma_{ij}$. As $e_i - e_j(t-T) \in \mathcal{L}_2$, we conclude that $\sigma_i - \sigma_j(t-T) \to 0$ and $\dot{\sigma}_i - \dot{\sigma}_j(t-T) \to 0$ according to Lemma 1.

Similarly, for the isolated agents, we get that $\sigma_i(t-T) = \sigma_i$ and $\dot{\sigma}_i(t-T) = \dot{\sigma}_i$ as $t \to +\infty$. For these isolated agents, taking agent s as an example, it will not always be isolated in the time interval $[t_k, t_{k+1})$. Assume that agent s becomes connected with agent j at time instant $t_k^{m_k}$. It can be concluded that $\sigma_s(t_k^{m_k}) = \sigma_s(t_k^{m_k} - T)$, also $\exists\varepsilon > 0$ and $\varepsilon \to 0$ for $\sigma_s(t_k^{m_k}+\varepsilon) = \sigma_s(t_k^{m_k})$, which is based on the continuity of $\sigma_s(t)$. It thus follows that $\sigma_s(t_k^{m_k}+\varepsilon-T) = \sigma_j(t_k^{m_k}+\varepsilon)$ and $\sigma_j(t_k^{m_k}+\varepsilon-T) = \sigma_s(t_k^{m_k}+\varepsilon)$. Then we can get that

$$\sigma_j(t_k^{m_k}+\varepsilon) = \sigma_j(t_k^{m_k}+\varepsilon-T) = \sigma_s(t_k^{m_k}+\varepsilon),$$

which implies that $\sigma_s(t_k^{m_k}) = \sigma_j(t_k^{m_k})$. Similarly, we get that $\dot{\sigma}_s(t_k^{m_k}) = \dot{\sigma}_j(t_k^{m_k})$ as $t_k^{m_k} \to +\infty$. From all the above analysis, we can conclude that

$$\begin{aligned}&\lim_{t\to\infty}|\sigma_i(t) - \sigma_j(t)| \to 0,\\ &\lim_{t\to\infty}|\dot{\sigma}_i(t) - \dot{\sigma}_j(t)| \to 0,\ i,j \in l.\end{aligned} \tag{16}$$

This completes the proof.

Remark 3 The attitude consensus control problems are also studied in [18] and [19] with communication time delays and dynamically changing topologies. However, the dynamically changing topologies are uniformly connected, which means that the topology is always connected and there is no isolated agent during each time interval $[t_k^m, t_k^{m+1})$.

Remark 4 For uniformly connected switching topologies, each agent keeps connected all the time, and therefore there are no isolated agents, but this does not hold for jointly connected switching topologies. In order to prevent the divergence of the isolated agents, we design different control laws for the connected and isolated agents respectively as (9). The control input for the isolated agents uses the difference information of the isolated agents between the current and the past.

Remark 5 By setting $T = 0$ and following the same lines of the proof of Theorem 1, it is straightforward to verify that the consensus problem (6) can also be solved for the time-delay free case.

3.2 Attitude consensus with varying time-delay and switching topologies

Denote $T(t)$ as the varying time delay from the jth agent to the ith agent. Suppose that the time delay is differentiable, bounded and satisfies

$$\dot{T}(t) \leqslant \varpi < 1, \tag{17}$$

where ϖ is a nonnegative constant. Also, we assume that $\ddot{T}(t)$ is bounded. Define a positive constant gain dependent on the maximum changing rate of delay as

$$d^2 < 1 - \varpi. \tag{18}$$

We design the input for the ith agent as (19), where $j \in \mathcal{N}_i$, $a_{ij}(t)$ is the i, j entry of the weighted adjacency matrix $\mathcal{A}_{\varrho(t)}$,

$$\eta_i = 1/\sum_{j=1}^{n} a_{ij}(t),\ \eta_j = 1/\sum_{i=1}^{n} a_{ji}(t),$$

$\gamma > 0$ is a constant, $e_i(t - T(t)) = \dot{\sigma}_i(t - T(t)) + c\sigma_i(t - T(t))$, and $T(t)$ is the varying communication time-delay.

$$u_i = \begin{cases} \omega_i^{\times} J_i\omega_i - \dfrac{J_iG_i^T}{p_i}[\dot{G}_i\omega_i + c\dot{\sigma}_i + \gamma\sum\limits_{j=1}^{n} a_{ij}(t)(\dfrac{\eta_i + d^2\eta_j}{2}e_i - d^2\eta_j e_j(t - T(t)))],\ i \in l_{\rm c}, \\ \omega_i^{\times} J_i\omega_i - \dfrac{J_iG_i^T}{p_i}[\dot{G}_i\omega_i + c\dot{\sigma}_i + \gamma(\dfrac{d^2+1}{2}e_i - d^2 e_i(t - T(t)))], \qquad i \in l_{\rm s}. \end{cases} \tag{19}$$

Combining (1) (5) (8) and (19), the derivative of e_i can be written as

$$\dot{e}_i = \begin{cases} -\gamma\sum\limits_{j=1}^{n} a_{ij}(t)(\dfrac{\eta_i + d^2\eta_j}{2}e_i - d^2\eta_j e_j(t - T(t))),\ i \in l_{\rm c}, \\ -\gamma[\dfrac{d^2+1}{2}e_i - d^2 e_i(t - T(t))], \qquad i \in l_{\rm s}. \end{cases} \tag{20}$$

Then we get the following result:

Theorem 2 Under Assumption 1, and with the control input as (19), the attitude consensus of system (1) is achieved as (6).

Proof Define a Lyapunov-Krasovskii function as $V(t) = \sum\limits_{j=1}^{n} V_i(t)$ for the system, where

$$V_i(t) = \frac{\gamma}{2}\int_{t-T(t)}^{t} e_i^{\rm T}(\tau)e_i(\tau){\rm d}\tau + \frac{1}{2}e_i^{\rm T}e_i. \tag{21}$$

Obviously, $V(t)$ is continuously differentiable in spite of the switching topologies, because $V(t)$ is independent of the switching topologies and $\dot{V}(t)$ exists.

According to the control input (19), we divide $V(t)$ into two parts, i.e., $V(t) = V_{\rm c}(t) + V_{\rm s}(t)$, where $V_{\rm c}(t) = \sum\limits_{i\in l_{\rm c}} V_i(t)$, $V_{\rm s}(t) = \sum\limits_{i\in l_{\rm s}} V_i(t)$.

Define $e_i^2(t - T(t)) \triangleq e_i^{\rm T}(t - T(t))e_i(t - T(t))$, then the derivative of $V_{\rm c}(t)$ is given by

$$\begin{aligned} &\dot{V}_{\rm c}(t) = \\ &\frac{\gamma}{2}\sum_{i\in l_{\rm c}}[e_i^{\rm T}e_i - (1 - \dot{T}(t))e_i^2(t - T(t))] + \sum_{i\in l_{\rm c}} e_i^{\rm T}\dot{e}_i = \\ &\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\eta_i\sum_{j=1}^{n} a_{ij}(t)[e_i^{\rm T}e_i - (1 - \dot{T}(t))e_i^2(t - T(t))] - \\ &\gamma\sum_{i\in l_{\rm c}} e_i^T\sum_{j=1}^{n} a_{ij}(t)[\frac{\eta_i + d^2\eta_j}{2}e_i - d^2\eta_j e_j(t - T(t))] \leqslant \\ &\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\eta_i\sum_{j=1}^{n} a_{ij}(t)[e_i^{\rm T}e_i - d^2 e_i^2(t - T(t))] - \\ &\gamma\sum_{i\in l_{\rm c}} e_i^{\rm T}\sum_{j=1}^{n} a_{ij}(t)[\frac{\eta_i + d^2\eta_j}{2}e_i - d^2\eta_j e_j(t - T(t))] = \\ &-\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\sum_{j=1}^{n} a_{ij}(t)\eta_j d^2[e_i^{\rm T}e_i + e_j^2(t - T(t))] + \\ &\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\sum_{j=1}^{n} a_{ij}(t)\eta_j d^2[2e_i^{\rm T}e_j(t - T(t))] = \\ &-\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\sum_{j=1}^{n} a_{ij}(t)\eta_j d^2[e_i - e_j(t - T(t))]^2. \end{aligned} \tag{22}$$

Similarly, the derivative of $V_{\rm s}(t)$ is given by

$$\begin{aligned} &\dot{V}_{\rm s}(t) = \\ &\frac{\gamma}{2}\sum_{i\in l_{\rm s}}[e_i^{\rm T}e_i - (1 - \dot{T}(t))e_i^2(t - T(t))] + \sum_{i\in l_{\rm s}} e_i^{\rm T}\dot{e}_i \leqslant \\ &\frac{\gamma}{2}\sum_{i\in l_{\rm s}}[e_i^{\rm T}e_i - d^2 e_i^2(t - T(t))] + \sum_{i\in l_{\rm s}} e_i^{\rm T}\dot{e}_i = \\ &-\frac{\gamma}{2}\sum_{i\in l_{\rm s}} d^2[e_i - e_i(t - T(t))]^2, \end{aligned} \tag{23}$$

where we have used the fact that

$$\begin{aligned} &\sum_{i\in l_{\rm c}}\sum_{j=1}^{n} a_{ij}(t)\eta_i e_i^{\rm T}(t - T(t))e_i(t - T(t)) = \\ &\sum_{i\in l_{\rm c}}\sum_{j=1}^{n} a_{ij}(t)\eta_j e_j^{\rm T}(t - T(t))e_j(t - T(t)). \end{aligned} \tag{24}$$

It is obvious that $V(t) \geqslant 0$ and $\dot{V}(t) \leqslant 0$, then it follows that $\lim\limits_{t\to\infty} V(t) = V(+\infty)$ exists. Similarly, we get that $e_i \in \mathcal{L}_\infty$ and $e_i - e_j(t - T(t)) \in \mathcal{L}_2$, thus $\sigma_i \in \mathcal{L}_\infty$, $\dot{\sigma}_i \in \mathcal{L}_\infty$ and $\dot{e}_i \in \mathcal{L}_\infty$. Then we conclude that $\ddot{V}(t)$ is bounded on each non-switching time interval. By invoking Lemma 2, we get $\lim\limits_{t\to\infty}\dot{V}(t) = 0$. Then, it follows that

$$\lim_{t\to\infty}\frac{\gamma}{2}\sum_{i\in l_{\rm c}}\sum_{j=1}^{n} d^2 a_{ij}(t)\eta_j|e_i - e_j(t - T(t))|^2 = 0 \tag{25}$$

and

$$\lim_{t\to\infty}\frac{\gamma}{2}\sum_{i\in l_{\rm s}}\sum_{j=1}^{n} d^2|e_i - e_i(t - T(t))|^2 = 0. \tag{26}$$

With similar analysis, we can get that $\sigma_i - \sigma_j(t - T(t)) \to 0$ and $\dot{\sigma}_i - \dot{\sigma}_j(t - T(t)) \to 0$ as $t \to \infty$ for the connected agents as well as the isolated agents.

Therefore we can conclude that

$$\begin{aligned}&\lim_{t\to\infty}|\sigma_i(t)-\sigma_j(t)|\to 0,\\&\lim_{t\to\infty}|\dot\sigma_i(t)-\dot\sigma_j(t)|\to 0,\ i,j\in l.\end{aligned}\tag{27}$$

This completes the proof.

Remark 6 In (19), if we set $\dot T(t)=0$ and $d=1$, then (19) equals to (9), which means that the constant time delay is a special case of the varying time delay.

Remark 7 In (19), we can also extend the varying time delay to the non-identical time delays. Define $T_i(t)$ as the communication delay transmitted from the ith agent to its neighbors, which also satisfies (17) and (18). If the control input is designed as (29) and the Lyapunov-Krasovskii function is chosen as

$$V(t)=\sum_{j=1}^{n}[\frac{\gamma}{2}\int_{t-T_i(t)}^{t}e_i^{\mathrm T}(\tau)e_i(\tau)d\tau+\frac{1}{2}e_i^{\mathrm T}e_i],\tag{28}$$

then the problem will be solved with similar process.

$$u_i=\begin{cases}\omega_i^\times J_i\omega_i-\dfrac{J_iG_i^{\mathrm T}}{p_i}[\dot G_i\omega_i+c\dot\sigma_i+\gamma\sum\limits_{j=1}^{n}a_{ij}(t)(\dfrac{\eta_i+d^2\eta_j}{2}e_i-d^2\eta_je_j(t-T_i(t)))], & i\in l_{\mathrm c},\\ \omega_i^\times J_i\omega_i-\dfrac{J_iG_i^{\mathrm T}}{p_i}[\dot G_i\omega_i+c\dot\sigma_i+\gamma(\dfrac{d^2+1}{2}e_i-d^2e_i(t-T_i(t)))], & i\in l_{\mathrm s}.\end{cases}\tag{29}$$

Remark 8 Since $\dot V(t)$ contains variable $\dot T(t)$, the expression of $\ddot V(t)$ is complicated and can be given as follow:

$$\begin{aligned}&\ddot V_{\mathrm c}(t)=\\&\frac{\gamma}{2}\sum_{i\in l_{\mathrm c}}\eta_i\sum_{j=1}^{n}a_{ij}(t)[2e_i^{\mathrm T}\dot e_i+\ddot T(t)e_i^2(t-T(t))-\\&2(1-\dot T(t))^2e_i^{\mathrm T}(t-T(t))\dot e_i(t-T(t))]-\\&\gamma\sum_{i\in l_{\mathrm c}}\sum_{j=1}^{n}a_{ij}(t)[(\eta_i+d^2\eta_j)e_i^T\dot e_i-d^2\eta_j\dot e_i^Te_j(t-\\&T(t))-d^2\eta_j(1-\dot T(t))e_i^T\dot e_j(t-T(t))],\end{aligned}\tag{30}$$

$$\begin{aligned}&\ddot V_{\mathrm s}(t)=\\&\frac{\gamma}{2}\sum_{i\in l_{\mathrm s}}[2e_i^{\mathrm T}e_i+\ddot T(t)e_i^2(t-T(t))-2(1-\\&\dot T(t))^2e_i^{\mathrm T}(t-T(t))\dot e_i(t-T(t))]-\\&\gamma\sum_{i\in l_{\mathrm s}}[(d^2+1)e_i^{\mathrm T}\dot e_i-\\&d^2\dot e_i^{\mathrm T}e_i(t-T(t))-d^2(1-\dot T(t))e_i^{\mathrm T}\dot e_i(t-T(t))].\end{aligned}\tag{31}$$

4 Numerical example

In this section, we present a numerical example with four rigid bodies to illustrate the effectiveness of our algorithm. Due to space limitation, we only consider the second case in this simulation, i.e., varying communication time delays coupled with jointly connected topologies. The dynamic equation of each follower is modeled by (1).

Fig.1 shows the switching topologies that characterize the interaction among the agents. In our simulation, we choose $a_{ij}=1,\ i=1,\cdots,4,\ j=1,\cdots,4,$ if agent j is a neighbor of agent i, and $a_{ij}=0$ otherwise. To make our problem interesting, we assume that the communication graph $\mathcal G_{\varrho(t)}$ associated with these rigid bodies is dictated by the following switching signal:

$$\varrho(t)=\begin{cases}1, & \lambda\tau\leqslant t<(\lambda+0.25)\tau,\\2, & (\lambda+0.25)\tau\leqslant t<(\lambda+0.5)\tau,\\3, & (\lambda+0.5)\tau\leqslant t<(\lambda+0.75)\tau,\\4, & (\lambda+0.75)\tau\leqslant t<(\lambda+1)\tau,\end{cases}\tag{32}$$

where $\tau=1$ s, $\lambda=0,1,2,\cdots$. The signal $\varrho(t)$ defines four fixed graphs $\mathcal G_i(i=1,2,3,4)$ as shown in Fig.1. Note that Assumption 1 is satisfied even though $\mathcal G_{\varrho(t)}$ is disconnected at any time $t\geqslant 0$.

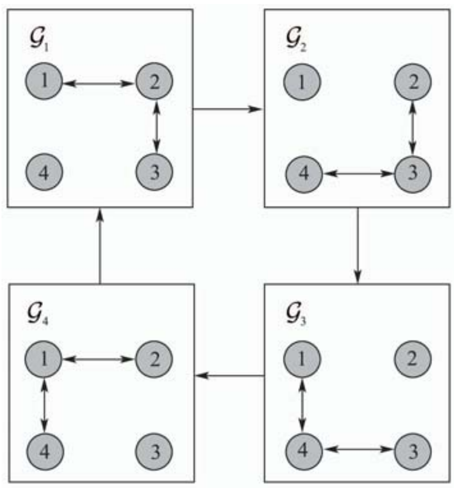

Fig. 1 The switching topologies that characterizes the interaction among the four agents

The initial attitudes of the four rigid bodies are set to be respectively, $\sigma_1=[4,3,4]^{\mathrm T}$, $\sigma_2=[0,3,1]^{\mathrm T}$, $\sigma_3=[5,1,2]^{\mathrm T}$, $\sigma_4=[-5,2,4]^{\mathrm T}$, and the initial angular velocities are set to be respectively, $\omega_1=[5,3,6]^{\mathrm T}$, $\omega_2=[3,2,4]^{\mathrm T}$, $\omega_3=[4,1,6]^{\mathrm T}$, $\omega_4=[1,5,2]^{\mathrm T}$. The communication delay is set to be $T(t)=0.5+0.1\sin t$, and the control parameters are chosen as $c=10$ and $\gamma=10$.

Fig.2 shows the attitude errors between agent 1 and agent 2, 3, 4 by using the control input (9). We see that the attitudes of the four agents converge to the same value. From Fig.3, we can see that the angular velocities of the four agents converge to the same value. Fig.4 shows the sliding-mode vector errors between agent 1 and agent 2, 3, 4. We can see that the sliding-mode vector errors $s_i-s_1(i=2,3,4)$ converge to zero, just as what we demonstrate in Theorem 2.

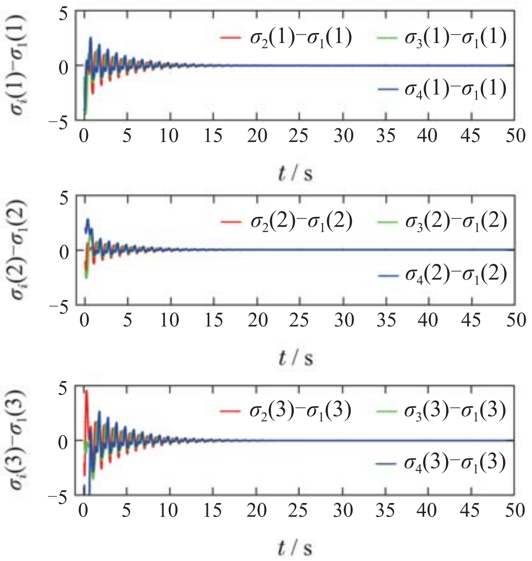

Fig. 2 The attitude error $\sigma_i - \sigma_1 (i = 2, 3, 4)$

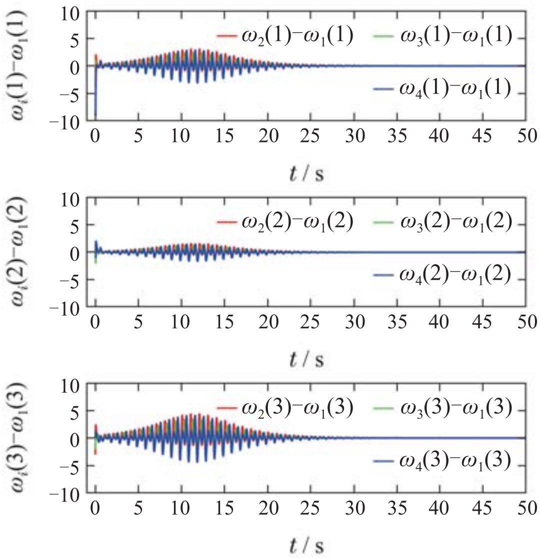

Fig. 3 The angular velocity error $\omega_i - \omega_1 (i = 2, 3, 4)$

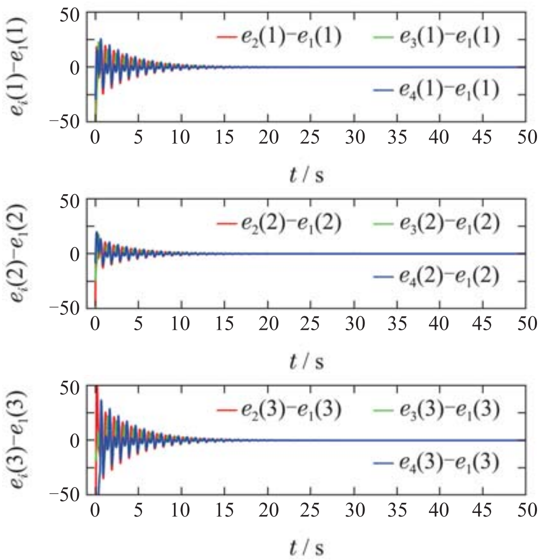

Fig. 4 The auxiliary vector error $e_i - e_1 (i = 2, 3, 4)$

Remark 9 The attitude and angular velocity states of the agents show the jumping phenomenon because of the switching topologies, whose period is synchronous with the switching signal $\varrho(t)$.

Remark 10 Consentability of the systems is not only related to the protocol design, but also effected by the average dwell time[32]. In this paper, we just assume that the average dwell time is sufficiently long, so that we only focus on the the design of the control algorithms.

5 Conclusions

In this study, we highlight the distributed attitude consensus problem for multiple rigid body systems under jointly connected switching topologies coupled with constant and varying communication time delays. Two distributed control algorithms are proposed based upon the distributed auxiliary vectors, and the consensus is achieved which renders the attitudes and the angular velocities of the agents to the same values. Numerical simulations are performed to support the theoretical analysis. Compared with previous algorithms, the proposed algorithms are able to achieve attitude consensus under jointly connected topologies, even coupled with time delays. However, the requirement on two-hop neighbors' information is conservative. Future work should be focused on the distributed attitude consensus problem in the case of directed topology together with heterogeneous time delays on the basis of the present study.

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