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至少有一个增长无源的子系统切换非线性系统的输出跟踪控制

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摘要: 本文利用平均驻留时间方法解决至少有一个增长无源的子系统的切换非线性系统的输出跟踪问题. 对于给定的无源率, 设计子系统控制器使得平均驻留时间变小. 所得到的平均驻留时间依赖于增长无源子系统的增长指数小时间范数可观性. 最后, 通过数值例子验证所提出方法的有效性.

关键词: 增长无源; 切换非线性系统; 输出跟踪; 平均驻留时间

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Output tracking control for switched nonlinear systems with at least an incrementally passive subsystem

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Abstract: This paper studies the output tracking control problem for a class of switched nonlinear systems with at least an incrementally passive subsystem via average dwell time method. For any given passivity rate, we can design feedback controllers for subsystems to make average dwell time dependent on incremental exponential small-time norm-observability property of incrementally passive subsystem get smaller and to solve the output tracking control problem for the switched nonlinear systems. A numeral example shows the effectiveness of the proposed method.

Key words: incremental passivity; switched nonlinear systems; output tracking; average dwell time

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1 Introduction

In the last decade, switched systems have received a great amount of attention due to the widespread application in many fields[1–2]. A switched control system is a special kind of hybrid dynamical systems consisting of a family of continuous-time subsystems and a rule that orchestrates the switching among them[3]. Many useful tools[4–10] have been developed to deal with such systems, such as the common Lyapunov function technique[4], the multiple Lyapunov functions method[5–6], the average dwell time approach[7–10] and so on.

On the other hand, the output tracking problem is to make the output of the plant, via a controller, track the given reference signal. The output track problem has been widely considered in aeronautics, robot control[11], flight control and so on. Compared with the stabilization problem, output tracking control is more challengeable. There have been many results for tracking control problem for nonlinear systems[12–15]. Due to the interaction between continuous dynamics and discrete dynamics, switched systems may have a very complicated behavior. Consequently, the output tracking control problem is more difficult and interesting for switched systems. In [16], a state-dependent switching law has been designed to achieve output tracking for the switched linear systems with time varying delays. For switched nonlinear systems, based on the sliding mode control method, the output tracking problem for cascade systems with external disturbance has also been addressed in [17]. In [9], the tracking control problem for switched linear time-varying delays systems with stabilizable and unstabilizable subsystems has been investigated via the average dwell time method.

Passivity theory can be traced back to the beginning

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of the 1970's. Passivity means that the energy dissipated inside a dynamic system is not larger than the energy supplied from outside. Passivity theory has been used to solve the output tracking problem[14–15, 18–20]. To study the energy relation between two trajectories, the concept of conventional passivity was extended to incremental passivity in [21]. Incremental passivity is also an input - output property originally proposed from an operator point of view in [22–23]. The incremental passivity concept in the state space form was proposed in [21]. The interconnection of two incrementally passive systems is still incrementally passive. Then, for incrementally passive system, an incrementally passive controller can be designed to drive the trajectories to converge to one another. Therefore, feedback control based on incremental passivity is particularly useful in studying output regulation problem for non-switched systems in [21]. Incremental passivity theory has been applied to the analysis of electrical circuits[24], and the synchronisation analysis problem of coupled oscillators[25]. The incremental passivity property is still useful for switched nonlinear systems. Incremental passivity theory and the incremental passivity-based output tracking problem for switched nonlinear systems using weak-storage functions and multiple supply rates were set up in [26], where the incremental passivity property for subsystems was not assumed. However, the activated subsystem is required to be incrementally passive.

In this paper, the output tracking control problem for a class of switched nonlinear systems is solved with the help of the incremental passivity property of subsystems via the average dwell time method. At least a subsystem is assumed to be incrementally passive. The results of this paper have three distinct features. Firstly, for any given passivity rate, the simple output feedback controllers of subsystems are designed to solve the output tracking control problem for the switched nonlinear systems. The average dwell time is dependent on incremental exponential small-time norm- observability property of incrementally passive subsystem and can get smaller by the controllers. Secondly, in this paper, some subsystems are allowed to be non-incrementally passive, when they are activated. Thirdly, the output tracking problem was solved under a given switched signal in [26]. Moreover, a state-dependent switching law is designed to render the switched systems incrementally passive. However, in this paper, the output tracking problem of switched nonlinear systems was solved under a class of switching signal with the average dwell time.

Notations $\mathbb{R}^n$: the n-dimensional Euclidean space. C^1 functions: continuously differentiable functions. $\mathbb{R}^+ : [0,+\infty)$. $i \in I_{\mathrm{P}}$: the ith subsystem is incremental passive. $i \in I_n \subseteq I - I_{\mathrm{P}}$: the ith subsystem is non-incrementally passive.

2 Incremental passivity

In this section, we will recall some basic theory on incrementally passivity for nonlinear system.

Definition 1[21] Consider a system

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x) \end{cases} \tag{1}$$

with state $x \in \mathbb{R}^n$, inputs $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^m$. System (1) is said to be incrementally passive if there exists a C^1 storage function $V(x, x') : \mathbb{R}^{2n} \to \mathbb{R}^+$ such that for any two inputs $u(t), u'(t)$ and any two solutions of system (1) $x(t), x'(t)$ corresponding to these inputs, the respective outputs $y(t) = h(x(t))$ and $y'(t) = h(x'(t))$ satisfy the following inequality.

$$\dot{V} \leqslant (y - y')^{\mathrm{T}}(u - u'). \tag{2}$$

Definition 2 System (1) is said to be exponentially incrementally passive if there exists constant $\lambda > 0$ such that the following inequality

$$\dot{V} + \lambda V \leqslant (y - y')^{\mathrm{T}}(u - u') \tag{3}$$

holds.

Remark 1 Notice that if $f(0) = 0,\ h(0) = 0$, then an incrementally passive (exponentially incrementally passive) system is also (exponentially) passive in the conventional sense with the storage function $\tilde{V}(x) = V(x, 0)$. The notion of exponential incremental passivity defined in this paper is more general than the notion of exponential passivity introduced in [30].

Definition 3 The nonlinear system

$$\dot{x} = f(x),\ y = h(x)$$

is said to be exponentially incremental small-time norm-observable with degree λ if there exists $\delta > 0$ such that when $\|y(t + s_0) - y'(t + s_0)\| \leqslant \delta$ holds for some $t \geqslant t_0,\ s_0 > 0$ and $0 < \tau \leqslant s_0$, we have

$$\|x(t + s) - x'(t + s)\| \leqslant c\mathrm{e}^{-\lambda s}\|x(t) - x'(t)\|.$$

Remark 2 In classical nonlinear passive systems, zero-state detectability is required for asymptotical stability. For switched systems, zero-state detectability is no longer effective due to switching. Several similar properties have been imposed to produce asymptotical stability, such as norm-observability[28], asymptotic zero-state detectability[29], exponential small-time norm-observability[8]. In order to get all solutions of the system converge to a bounded solution, the exponentially incremental small-time norm-observable is needed.

Lemma 1 Suppose that the system (1) with $u(t) = 0$ has a bound solution. $\bar{x}(t)$ output $y = h(\bar{x})$ for $t \geqslant t_0$. Then, the inequalities (2)–(3) are equivalent to the following conditions (4)–(5), respectively.

$$\begin{cases} \dfrac{\partial V}{\partial x} f(x) + \dfrac{\partial V}{\partial \bar{x}} f(\bar{x}) \leqslant 0, \\ \dfrac{\partial V}{\partial x} g(x) = h(x) - h(\bar{x}), \end{cases} \tag{4}$$

$$\begin{cases} \dfrac{\partial V}{\partial x} f(x) + \dfrac{\partial V}{\partial \bar{x}} f(\bar{x}) \leqslant -\lambda V, \\ \dfrac{\partial V}{\partial x} g(x) = h(x) - h(\bar{x}). \end{cases} \tag{5}$$

Proof Since system (1) is incrementally passive, we get

$$\dot{V} = \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x'} f(x') + \frac{\partial V}{\partial x} g(x)u + \frac{\partial V}{\partial x'} g(x')u' \leqslant (y - y')^{\mathrm{T}}(u - u'). \tag{6}$$

Substituting $u'(t) = 0,\ x' = \bar{x}(t),\ y' = h(\bar{x})$ yields for $\forall u$,

$$\dot{V} = \frac{\partial V}{\partial x} f(x) + \frac{\partial V}{\partial x'} f(\bar{x}) + \frac{\partial V}{\partial x} g(x)u \leqslant (y - y')^{\mathrm{T}} u. \tag{7}$$

Obviously, the condition (7) is equivalent to condition (4).

Similarly to the proof of (4), the equality (5) holds. Note that by taking $u' = 0,\ \bar{x} = 0,\ y' = h(\bar{x}) = 0$, the above condition satisfies the Hill-Moylan condition[27].

QED.

3 Problem formulation

Consider a switched nonlinear system described by

$$\begin{cases} \dot{x} = f_{\sigma(t)}(x) + g_{\sigma(t)}(t) u_{\sigma(t)}, \\ y = h_{\sigma(t)}(x) \end{cases} \tag{8}$$

with state $x \in \mathbb{R}^n$, a piecewise constant function depending on time $\sigma(t) : [0, \infty) \to I = \{1, 2, \cdots, M\}$, called the switching signal. It is assumed to be switching a finite number of times in any finite time interval so as to rule out Zeno behavior[4]. $u_i \in \mathbb{R}^m$ and $h_i(x) \in \mathbb{R}^m$ are the input vector and the output vector of the ith subsystem, respectively. $f_i(x), g_i(x), h_i(x)$ are smooth functions. Corresponding to the switching signal, we have the switching sequence

$$\Sigma = \{x_0; (i_0, t_0), (i_1, t_1), \cdots, (i_k, t_k), \cdots, |i_k \in I,\ k \in \mathbb{N}\}, \tag{9}$$

where t_0 is the initial time, x_0 is the initial state, and $\mathbb{N}$ is the set of nonnegative integers. When $t \in [t_k, t_{k+1})$, $\sigma(t) = i_k$, that is, the i_kth subsystem is active. In this paper, we focus on the output tracking control problem for the system (8). In order to study the solvability of the output tracking problem, we make the following assumption.

Assumption 1 Let $Y \subset \mathbb{R}^m$ be a domain of reference signal. For any $y_{\mathrm{r}}(t) \in Y,\ u_i(t) = 0,\ t \geqslant t_0$, there exist unique a bound solution $\bar{x}(t)$ such that $y_{\mathrm{r}}(t) = h_i(\bar{x}(t))$.

The output tracking control problem for the switched systems can be formulated as follows:

Design controllers u_i for each subsystem such that, for given a family of switching signals $\sigma(t)$.

1) The state of the closed-loop system (8) is globally bounded.

2) $\lim\limits_{t \to \infty} (y(t) - y_{\mathrm{r}}(t)) = 0$.

4 Output tracking of switched nonlinear systems

In this section, we will design controller for each subsystem to solve the output tracking problem for any given average dwell time with the help of the incremental passivity property of subsystems.

In order to solve the output tracking control problem, at least one subsystem is assumed to be incrementally passive. Here we do not need incremental passivity of all subsystems. For convenience, the subsystems of (8) is classified into two groups. The system (8) is incremental passive for $i \in I_{\mathrm{P}} \subseteq I$, and non-incrementally passive for $i \in I_n \subseteq I - I_{\mathrm{P}}$. Also, the activation time ratio of incrementally passive subsystems and non-incrementally passive subsystems plays a crucial role. Therefore, the incremental passivity rate is defined as follows.

Definition 4[8] For any $0 \leqslant T_1 < T_2$, let $T_{p[T_1,T_2]}$ denote the total activation time of the incremental passive subsystems during $[T_1, T_2]$. Then $r_{p[T_1,T_2]} = \dfrac{T_{p[T_1,T_2]}}{T_2 - T_1}$ is called the incremental passivity rate of the switched system (8). Obviously, $0 < r_{p[T_1,T_2]} \leqslant 1$.

The notion of average dwell time is introduced in [10] as follows.

Definition 5[10] For a switching signal $\sigma(t)$ and any $t > \tau > 0$, let $N_\sigma(\tau, t)$ be the switching numbers of $\sigma(t)$ over the interval (τ, t). If

$$N_\sigma(\tau, t) \leqslant N_0 + \frac{t - \tau}{\tau_{\mathrm{a}}}$$

holds for $N_0 \geqslant 0,\ \tau_{\mathrm{a}} > 0$, then τ_{a} and N_0 are called the average dwell time and the chatter bound, respectively.

Now, we are in a position to give solvability conditions on the output tracking problem for switched systems with incrementally passive and non-incrementally passive subsystems.

Theorem 1 Suppose that Assumption 1 holds and there exist C^1 nonnegative storage function $V_i(x, x')$ and constants $\alpha_1 > 0, \alpha_2 > 0$, such that for $i, j \in I$.

1) $\alpha_1 \|x - x'\|^2 \leqslant V_i(x, x') \leqslant \alpha_2 \|x - x'\|^2$.

2) The activation time of the incremental passive subsystems during $[T_1, T_2]$ satisfies with $T_{P[T_1,T_2]} \geqslant r(T_2 - T_1) - T_0$ for some constant $T_0 \geqslant 0,\ r \in (0, 1]$.

3) For $i \in I_n$, there exists $\lambda_{i2} > 0$ such that the i-th subsystem satisfies

$$\frac{\partial V_i}{\partial x} f(x) + \frac{\partial V_i}{\partial \bar{x}} f(\bar{x}) \leqslant \lambda_{i2} V_i(x, \bar{x}). \tag{10}$$

4) For $i \in I_{\mathrm{P}}$, the subsystem is exponentially in-

cremental small-time norm-observa-ble with degree

$$\bar{\lambda} \geqslant \frac{\lambda_1}{2},\ c \leqslant \sqrt{\frac{\alpha_1}{\alpha_2}}$$

under $u_i = 0$, where

$$\lambda^* = \lambda_1 r - \lambda_2(1-r) - \frac{\ln\mu}{\tau_a},\ \mu = \frac{\alpha_2}{\alpha_1}$$

for some constants $\lambda^* > 0,\ \lambda_1 > 0,\ \lambda_2 = \max_{i\in I_n}\{\lambda_{i2}\}$. Then, the output tracking control problem is solved with the average dwell time $\tau_a > \ln\mu/(\lambda_1 r - \lambda_2(1-r))$ and the controllers are given by

$$u_i = \begin{cases} -k_i(x,\tau_a,r)(L_{g_i}V_i)^T, & i \in I_P, \\ 0, & i \in I_n, \end{cases} \tag{11}$$

where

$$k_i(x,\tau_a,r) = \begin{cases} \lambda_1(\|L_{g_i}V_i\|^2)^{-1}V_i(x,\bar{x}), & \|L_{g_i}V_i\| > \delta, \\ 0, & \|L_{g_i}V_i\| \leqslant \delta. \end{cases}$$

Proof Define $S = \{t : \|L_{g_i}V_i(t))\| \leqslant \delta,\ i \in I_p\}$. We now split the proof into two cases of $S = \varphi$ and $S \neq \varphi$.

Case 1 $S = \varphi$. When $\sigma(t) = i$, differentiating $V_i(x, x')$ along the trajectory of the switched system (8) gives

$$\dot{V}_i = \frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial x'} f_i(x') + \frac{\partial V_i}{\partial x} g_i(x)u_i + \frac{\partial V_i}{\partial x'} g_i(x')u_i'. \tag{12}$$

Substituting $u_i'(t) = 0,\ x'(t) = \bar{x}(t)$ yields

$$\dot{V}_i = \frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \bar{x}} f_i(\bar{x}) + \frac{\partial V_i}{\partial x} g_i(x)u_i. \tag{13}$$

For $i \in I_P$, according to Lemma 1, there exist C^1 storage function $V_i(x,x')$ such that (4) holds for any two inputs $u_i(t), u_i'(t)$.

Substituting (4) and the controllers (11) into (13) yields for $i \in I_P$,

$$\dot{V}_i \leqslant \frac{\partial V_i}{\partial x} g_i(x)\, u_i = -L_{g_i}V_i\lambda_1(\|L_{g_i}V_i\|^2)^{-1}V_i(L_{g_i}V_i)^T = -\lambda_1 V_i. \tag{14}$$

Similarly, for $i \in I_n$, it follows from (11) and $u_i(t) = 0$ that

$$\dot{V}_i \leqslant \lambda_2 V_i, \tag{15}$$

where $\lambda_2 = \max_{i\in I}\{\lambda_{i2}\}$.

Then, using the differential inequality theory for (14)–(15) gives that for $\forall t > s \geqslant t_0$.

$$V_i(x(t),\bar{x}(t)) \leqslant e^{-\lambda_i(t-s)}V_i(x(s),\bar{x}(s)), \tag{16}$$

$$\lambda_i = \begin{cases} \lambda_1, & \text{when } i \in I_P, \\ -\lambda_2, & \text{when } i \in I_n. \end{cases}$$

Without loss of generality, for any given $t > t_0,\ t \in [t_k, t_{k+1})$, and thus $N_\sigma(t_0,t) = k$. By (16), we get

$$\begin{cases} V_{i_{j-1}}(x(t_j),\bar{x}(t_j)) \leqslant \\ e^{-\lambda_{i_j}(t_j-t_{j-1})}V_{i_{j-1}}(x(t_{j-1}),\bar{x}(t_{j-1})), \\ j = 1,2,\cdots,k-1, \\ V_{i_k}(x(t),\bar{x}(t)) \leqslant e^{-\lambda_{i_k}(t-t_k)}V_{i_k}(x(t_k),\bar{x}(t_k)). \end{cases} \tag{17}$$

Since

$$V_i(x,x') \leqslant \mu V_j(x,x'),\ \mu = \frac{\alpha_2}{\alpha_1} \geqslant 1$$

hold according to condition 1. We obtain by induction that

$$\begin{aligned} &V_{\sigma(t)}(x(t),\bar{x}(t)) = V_{i_k}(x(t),\bar{x}(t)) \leqslant \\ &\mu^k e^{-\lambda_{i_k}(t-t_k)-\sum_{j=1}^{k}\lambda_{i_{j-1}}(t_j-t_{j-1})}V_{i_0}(x(t_0),\bar{x}(t_0)) \leqslant \\ &\mu^{N_\sigma(t_0,t)}e^{-\lambda_1 T_{p[\tau,t]}+\lambda_2 T_{n[\tau,t]}}V_{i_0}(x(t_0),\bar{x}(t_0)) \leqslant \\ &e^{N_\sigma(t_0,t)\ln\mu-\lambda_1 T_{p[t_0,t]}+\lambda_2 T_{n[t_0,t]}}V_{i_0}(x(t_0),\bar{x}(t_0)). \end{aligned} \tag{18}$$

Therefore, from the definitions of the average dwell time and the passivity rate, we have

$$\begin{aligned} &(N_0 + \frac{t-t_0}{\tau_a})\ln\mu - \lambda_1 T_{p[t_0,t]} + \lambda_2 T_{n[t_0,t]} \leqslant \\ &(N_0 + \frac{t-t_0}{\tau_a})\ln\mu - \lambda_1 r(t-t_0) + \\ &(\lambda_1+\lambda_2)T_0 + \lambda_2(1-r)(t-t_0) \leqslant \\ &N_0\ln\mu - (\lambda_1 r - \lambda_2(1-r) - \frac{\ln\mu}{\tau_a}) \\ &(t-t_0) + (\lambda_1+\lambda_2)T_0 = \\ &N_0\ln\mu - \lambda^*(t-t_0) + (\lambda_1+\lambda_2)T_0. \end{aligned} \tag{19}$$

Substituting (19) into (18) gives rise to

$$\begin{aligned} &V_{\sigma(t)}(x(t),\bar{x}(t)) \leqslant \\ &e^{N_0\ln\mu-\lambda^*(t-t_0)+(\lambda_1+\lambda_2)T_0}V_{\sigma(t_0)}(x(t_0),\bar{x}(t_0)). \end{aligned} \tag{20}$$

It is easy to deduce from $\alpha_1\|x-x'\|^2 \leqslant V_i(x,x') \leqslant \alpha_2\|x-x'\|^2$ that

$$\begin{aligned} &\alpha_1(\|x(t)-\bar{x}(t)\|) \leqslant \\ &e^{N_0\ln\mu-\lambda^*(t-t_0)+(\lambda_1+\lambda_2)T_0}\alpha_2(\|x(t_0)-\bar{x}(t_0)\|), \\ &\|x(t)-\bar{x}(t)\| \leqslant \\ &\alpha_1^{-1}\cdot\alpha_2 e^{N_0\ln\mu-\lambda^*(t-t_0)+(\lambda_1+\lambda_2)T_0}(\|x(t_0)-\bar{x}(t_0)\|). \end{aligned} \tag{21}$$

When $\lambda^* > 0,\ t \to \infty$, the state $x(t)$ converge to $\bar{x}(t)$, namely, $\|x(t)-\bar{x}(t)\| \to 0,\ t \to \infty$. Therefore, the state of the closed-loop system (8) is globally bounded and $h_i(x(t)) \to h_i(\bar{x}(t)),\ t \to \infty$.

Case 2 $S \neq \varphi$.

For $i \in I_P$, since $h_i(x)$ is a continuous function, $L_{g_i}V_i(x)$ is continuous when the corresponding subsystem is active. Without loss of generality, we suppose that the set

$$\{t : \|L_{g_i} V_i(x(t))\| \leqslant \delta\} = [t_{i_1}, t'_{i_1}] \cup [t_{i_2}, t'_{i_2}] \cup \cdots \subset [t_0, t] \text{ for } \delta > 0.$$

Since each incremental passive subsystem is incrementally exponential small-time norm-observability, we have

$$\begin{aligned}
&\|x(t'_{i_k}) - \bar{x}(t'_{i_k})\| \leqslant \\
&c\,\mathrm{e}^{-\lambda_1(t'_{i_k} - t_{i_k})}\|x(t_{i_k}) - \bar{x}(t_{i_k})\| \leqslant \\
&\sqrt{\frac{\alpha_1}{\alpha_2}}\mathrm{e}^{-\bar{\lambda}(t'_{i_k} - t_{i_k})}\|x(t_{i_k}) - \bar{x}(t_{i_k})\|, \\
&V_i(x(t'_{i_k}), \bar{x}(t'_{i_k})) \leqslant \\
&\mathrm{e}^{-2\bar{\lambda}(t'_{i_k} - t_{i_k})} V_i(x(t_{i_k}), \bar{x}(t_{i_k})) \leqslant \\
&\mathrm{e}^{-\lambda_1(t'_{i_k} - t_{i_k})} V_i(x(t_{i_k}), \bar{x}(t_{i_k})).
\end{aligned} \tag{22}$$

According to condition (17), when the incrementally passive subsystem is active and $\|L_{g_i} V_i\| \geqslant \delta$, for $t \in [t'_{i_k}, t_{i_k+1}]$, $k = 1, 2, \cdots$, we have

$$\begin{aligned}
&V_i(x(t_{i_k+1}), \bar{x}(t_{i_k+1})) \leqslant \\
&\mathrm{e}^{-\lambda_1(t_{i_k+1} - t'_{i_k})} V_i(x(t'_{i_k}), \bar{x}(t'_{i_k})).
\end{aligned} \tag{23}$$

Then,

$$\begin{aligned}
&V_i(x(t_{i_k+1}), \bar{x}(t_{i_k+1})) \leqslant \\
&\mathrm{e}^{-\lambda_1(t_{i_k+1} - t_{i_k})} V_i(x(t_{i_k}), \bar{x}(t_{i_k})).
\end{aligned} \tag{24}$$

Using the similar method of the proof for the Case 1, we can obtain that the state of the closed-loop system (8) is globally bounded and $\lim\limits_{t\to\infty}(y(t) - y_{\mathrm{r}}(t)) = 0$. This completes the proof of Theorem 1. QED.

Remark 3 The passivity rate is larger than the constant $\dfrac{\lambda_2}{\lambda_2 + \lambda_1}$. This means that incrementally passive subsystems have to be active in some time interval and non-incrementally-passive subsystems are allowed to be active, which is not allowed in [26]. When $\lambda_2 > \lambda_1$, the total activation time of incrementally-passive subsystems is longer than the total activation time of non-incrementally-passive subsystems.

Remark 4 If we suppose that at least an exponentially incrementally passive subsystem is required, the property of incrementally exponential small-time norm-observability is not needed and the controller gain gets smaller.

Remark 5 For each active non-incrementally passive subsystem, the energy is increasing and V_i is required to change no faster than an exponential rate of increase, as shown in (11). We can design controllers of the corresponding incrementally passive subsystems and regulate the gains to compensate the change of the energy function when non-incrementally passive subsystems are active. Therefore, at least one incrementally passive subsystem is needed. On the other hand, the gains of controllers are designed to be larger than a constant that is determined by the average dwell time and the passivity rate. This means that, given an average dwell time τ_α, we can design feedback controllers with respect to τ_α for subsystems to stabilize the switched system.The larger the gains, the faster the convergence rate of the state.

Corollary 1 Consider a switched system of the form

$$\begin{cases} \dot{x} = F_{\sigma(t)}(x) + B_{\sigma(t)} u_{\sigma(t)}, \\ \tilde{y} = Cx, \end{cases} \tag{25}$$

where $F_i(x)$ are C^1 in x, B_i and C are constant matrices of appropriate dimensions. Suppose that for $u_\sigma(t) = 0$, the system (25) has a bound solution $\bar{x}(t)$ with output $\bar{\tilde{y}} = C\bar{x} = y_{\mathrm{r}}(t)$ for $t \geqslant t_0$, and there exist positive definite matrices P_i, such that $i, j \in I$.

1) The activation time of the incremental passive subsystems during $[T_1, T_2]$, $T_{P[T_1,T_2]} \geqslant r(T_2 - T_1) - T_0$ is satisfied for constant $T_0 \geqslant 0,\ r \in (0, 1]$.

2) For $i \in I_n$, there exists positive definite matrices Q_i such that the subsystem with $u_i = 0$ satisfy

$$P_i \frac{\partial F_i}{\partial x}(x) + \frac{\partial F_i^{\mathrm{T}}}{\partial x}(x) P_i \leqslant Q_i. \tag{26}$$

3) For $i \in I_{\mathrm{P}}$, the subsystem is exponentially incremental small-time norm-observable with degree

$$\bar{\lambda} \geqslant \frac{\lambda_1}{2},\ c \leqslant \sqrt{\frac{\alpha_1}{\alpha_2}}$$

under $u_i = 0$, where $\lambda^* = \lambda_1 r - \lambda_2(1 - r) - \ln\mu/\tau_{\mathrm{a}}$ for some constants

$$\lambda_2 = \frac{\max\limits_{i\in I}\{\lambda_{\max}(Q_i)\}}{\min\limits_{i\in I}\{\lambda_{\min}(P_i)\}},$$

$$\mu = \frac{\max\limits_{i\in I}\{\lambda_{\max}(P_i)\}}{\min\limits_{i\in I}\{\lambda_{\min}(P_i)\}},\ \lambda^* > 0,\ \lambda_1 > 0,$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimal and the maximal eigenvalues of a symmetric matrix. Then, the output tracking control problem is solved under the average dwell time $\tau_{\mathrm{a}} > \ln\mu/(\lambda_1 r - \lambda_2(1-r))$ for some constant $\lambda_2 > 0$ and the controllers are given by

$$u_i(x) = \begin{cases} -k_i(x, \tau_{\mathrm{a}}, r)(L_{g_i} V_i)^{\mathrm{T}}, & i \in I_{\mathrm{P}}, \\ 0, & i \in I_n, \end{cases} \tag{27}$$

where

$$\begin{aligned}
&k_i(x, \tau_{\mathrm{a}}, r) = \lambda_1(\|L_{g_i} V_i\|^2)^{-1} V_i(x, \bar{x}), \\
&\|L_{g_i} V_i\| > \delta,\ k_i(x, \tau_{\mathrm{a}}, r) = 0,\ \|L_{g_i} V_i\| \leqslant \delta, \\
&V_i(x, \bar{x}) = \frac{1}{2}(x - \bar{x})^{\mathrm{T}} P_i (x - \bar{x}), \\
&L_{g_i} V_i = P_i B_i (x - \bar{x}).
\end{aligned}$$

Proof When $\sigma(t) = i$, differentiating $V_i(x, x')$ along the trajectory of the switched system (26) gives

$$\begin{aligned}
\dot{V}_i = &(x - x')^{\mathrm{T}} P_i (F_i(x) - F_i(x')) + \\
&(x - x')^{\mathrm{T}} P_i B_i (u_i - u'_i).
\end{aligned} \tag{28}$$

Substituting $u_i'(t)=0,\ x'(t)=\bar{x}(t)$, yields

$$\dot{V}_i=(x-\bar{x})^{\mathrm{T}}P_i(F_i(x)-F_i(\bar{x}))+ (x-\bar{x})^{\mathrm{T}}P_iB_iu_i. \tag{29}$$

Similar to [21], according to the mean value theorem, we obtain

$$(x-\bar{x})^{\mathrm{T}}P_i(F_i(x)-F_i(\bar{x}))= \frac{1}{2}(x-\bar{x})^{\mathrm{T}}J_i(\xi)(x-\bar{x}), \tag{30}$$

where

$$J_i(x)=P_i\frac{\partial F_i}{\partial x}(x)+\frac{\partial F_i^{\mathrm{T}}}{\partial x}(x)P_i,$$

ξ is some point between x_1 and x_2. According to (27), we obtain

$$\dot{V}_i\leqslant\frac{1}{2}(x-\bar{x})^{\mathrm{T}}Q_i(x-\bar{x})\leqslant\lambda_2V_i. \tag{31}$$

According to Theorem 1, the output tracking problem is solved. QED.

Remark 6 For $i\in I_{\mathrm{P}}$, there exists positive definite matrices Q_i' such that the subsystem with $u_i=0$ satisfied

$$P_i\frac{\partial F_i}{\partial x}(x)+\frac{\partial F_i^{\mathrm{T}}}{\partial x}(x)P_i\leqslant -Q_i'.$$

The property of incrementally exponential small-time norm-observability is not needed.

5 Numerical example

In this section, a numerical example will be presented to demonstrate the potential and validity of our developed theoretical results.

Consider systems (8) consisting of two subsystems described by

subsystem 1

$$\begin{cases}\dot{x}_1=-x_1\left(x_1^2+4\right)+\dfrac{1}{2}x_2+4+u_1,\\ \dot{x}_2=\dfrac{1}{2}x_1-x_2+\dfrac{3}{2}+\dfrac{1}{3}u_1,\end{cases}$$

subsystem 2

$$\begin{cases}\dot{x}_1=x_1+x_2-3+0.5u_2,\\ \dot{x}_2=-2x_1+x_2+u_2\end{cases}$$

with the common output $y=x_1+x_2$. $y_{\mathrm{r}}=3$ is reference signal.

We choose the storage functions as

$$S_1(x,\hat{x})=\frac{1}{2}(x-\hat{x})^{\mathrm{T}}P_1(x-\hat{x}),$$
$$S_2(x,\hat{x})=\frac{1}{2}(x-\hat{x})^{\mathrm{T}}P_2(x-\hat{x}),$$

where

$$P_1=\begin{bmatrix}1&0\\0&3\end{bmatrix},\ P_2=\begin{bmatrix}2&0\\0&1\end{bmatrix}.$$

Differentiating S_i gives

$$\dot{S}_1\leqslant -1.8S_1+(u_1-\hat{u}_1)^{\mathrm{T}}(y-\hat{y}),$$
$$\dot{S}_2\leqslant 2.1S_2+(u_2-\hat{u}_2)^{\mathrm{T}}(y-\hat{y}).$$

Therefore, subsystem 1 is incrementally passive and subsystem 2 is non-incrementally passive. According to Theorem 1, the output track problem is solved by $u_i=-(y-y_{\mathrm{r}})$ under the average dwell

$$\tau_{\mathrm{a}}\geqslant\frac{\ln\mu}{\lambda_{\mathrm{P}}\gamma_{\mathrm{P}}-\lambda_n(1-\gamma_{\mathrm{P}})},\ \lambda_{\mathrm{P}}=1.8,\ \lambda_n=2.1.$$

Moreover, let $\mu=3$ and the incremental passivity rate $\gamma_{\mathrm{P}}=0.8$. Then, the average dwell time is chosen as $\tau_{\mathrm{a}}=1.5>1.17$. The simulation results are depicted in Figs. 1–3 for the initial state $x(0)=(11.7,-24.7)$. It can be seen from Fig.1 and Fig.3 that the global output track problem is solvable under a class of switching signal σ with average dwell time $\tau_{\mathrm{a}}=1.5$. The switching signal is given by Fig.3. Thus, the simulation results well illustrate the theory presented.

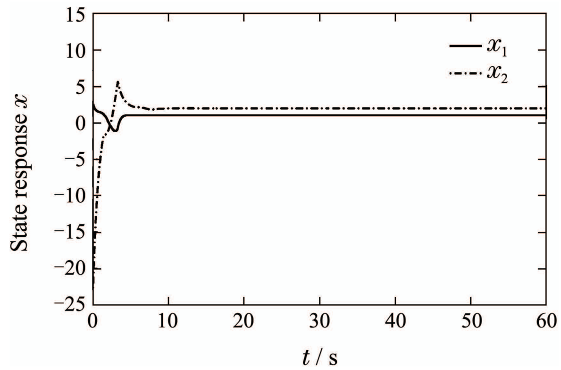

Fig. 1 State response of the switched system

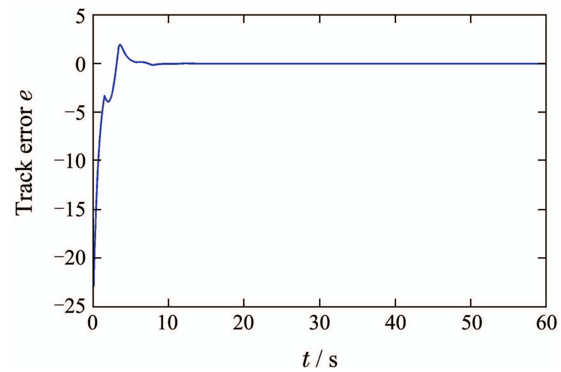

Fig. 2 The track error of the switched system

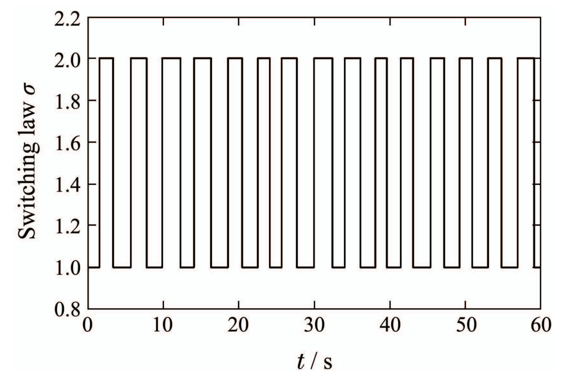

Fig. 3 The switching signal of the switched system

6 Conclusions

This paper investigates the output tracking control problem for a class of switched nonlinear systems using incremental passivity concept via average dwell time method. For any given passivity rate, we can design feedback controllers of subsystems which make average dwell time dependent on incremental exponential small-time norm- observability property of incrementally passive subsystem get smaller to solve the output tracking control problem for the switched nonlinear systems.

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