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具有群集行为的时变函数分布式优化

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摘要: 本文对具有群集行为的连续时间多智能体系统的优化问题进行了研究. 考虑具有二阶动力学的多智能体系统, 每个智能体都具有一个局部的时变代价函数. 本文的目标是仅仅依靠局部信息交流使得多智能体在运动的过程中保持连通性、避免碰撞、总体代价函数最小. 为此本文设计了一种具有群集行为的控制协议, 该协议仅仅依赖于自己和邻居的速度. 可以证明在该控制协议作用下, 所有智能体在保持连通、避免碰撞的同时, 速度能够跟踪上最优速度. 最后, 通过一个仿真来说明本文的结果.

关键词: 分布式优化; 群集; 多智能体系统; 保持连通

中图分类号: TP13, TP18A　　　**文献标识码**: A

Distributed velocity optimization of time-varying functions with flocking behavior

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Abstract: This paper studies optimization problem for continuous-time multi-agent systems with flocking behavior. Multi-agents with second-order dynamics are considered. Each agent is equipped with a local time-varying cost function which is known only to an individual agent. The objective is to make multi-agents' velocities minimize the sum of local functions by local interaction, while avoiding collision and preserving connectivity. A distributed protocol with flocking behavior is presented, in which each agent depends only on its own velocity and neighbor's velocities. It can be proved that under the control protocol, all agents remain connected and avoid collisions while the velocity of the agents tracks the optimal velocity.

Key words: distributed optimization; flocking; multi-agent systems; connectivity-preserving

1 Introduction

Flocking of multi-agents has attracted much attention in the literature. The aim is that a group of agents move with local interaction while preserving connectivity, avoiding collisions, and having the same velocities. For decades, more and more researchers devote themselves to study flocking[1–8]. Olfati-Saber[2] proposed a framework to design and analyse a scalable flocking algorithms. Tanner et al.[3] presented a control rule that makes multi-agents realize flocking motion in both fixed and switching networks. Su et al.[4] gave a preserving connectivity flocking algorithm using only position measurements for multi-agents. Recently, flocking with more complicated dynamics was researched[5–8]. Su et al.[5] investigated the adaptive flocking problem for multi-agent systems with local Lipschitz nonlinearity. Wang[6] investigated the flocking problem with heterogeneous nonlinear dynamics. Yang and Zhang[7] investigated the flocking with nonlinear inner-coupling functions. Ghapani[8] investigated a leader-follower flocking problem for networked Lagrange systems having uncertain parameters. In the aforementioned researches, optimization problem has not been taken into account. However, in many applications, it is necessary for agents to cooperatively optimize a certain standard.

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In recent years, the distributed optimization has been intensively researched. The aim of distributed optimization is to minimize the sum of convex functions through information exchange with its neighbors. Results of distributed optimization include two categories: discrete-time models and continuous-time models. A distributed optimization method using the local sub-gradient and local information over a time-varying topology was presented for unconstrained distributed optimization in [9]. Nedic proposed a distributed projected consensus algorithm and researched its convergence properties over constraint sets[10]. Some extended or modified models were also presented for constrained distributed optimization[11–12]. Continuous-time algorithm for distributed optimization was introduced in [13–14], in which the topology was assumed to be undirected. Yi[13] proposed a distributed Stochastic sub-gradient algorithm for distributed optimization with random sleep scheme. Article [14] proposed a continuous-time algorithm and established the convergence analysis by LaSalle's Invariance Principle for strongly connected and weight-balanced digraphs. Lin[15] studied the optimization problem with adaptivity and finite time convergence for single-integrator agents. Wang[16] studied the distributed optimization problem for a class of nonlinear multi-agent systems in the presence of external disturbances. Zhang[17] studied the gradient-based optimization design for the second-order agent dynamics with a general optimization setup and gave a Lyapunov-based method with some modification of existing techniques. A second-order multi-agent network for bound-constrained distributed optimization was proposed in [18]. Zhang[19] discussed the distributed optimal coordination problem for multi-agent systems with the agents in the form of Euler-Lagrangian(EL) dynamics. In article [20], a time-varying distributed convex optimization problem was studied for continuous-time multi-agent systems, where the objective is to minimize the sum of the local time-varying cost functions. With the interest in decentralized architectures and motivated by the problem of distributed convex optimization, a distributed version of online optimization is proposed in [21–23]. Yan et al. in [22] introduced a decentralized online optimization based on the sub-gradient method in which the agents interact over a weighted strongly connected directed graph. The suggested protocol in [23] works on jointly connected weight-balanced digraphs.

There are many results for distributed optimization of multi-agents, though flocking was not taken into account. So in this paper, we consider the optimization problem of time-varying cost function with flocking behavior. Each agent is equipped with a local time-varying cost function and second order dynamics. The objective is to make the velocity of the agents track the optimal velocity which minimizes the sum of time-varying cost functions through local interaction, meanwhile, the agents will preserve connectivity and avoid collision between agents.

We organize the paper as follows. In Section 2, notations and some basic concepts used in this paper are introduced. In Section 3, a distributed optimization scheme with flocking behavior and time-varying functions is designed. In Section 4, simulation result is presented to substantiate the theoretical results. Finally, conclusions are provided in Section 5.

2 Notations and preliminaries

Notations and concepts from graph theory and convex functions are given, in this section.

Denote $\mathbf{1}_m = (1\ 1\ \cdots\ 1)^{\mathrm{T}}$, $\mathbf{0}_m = (0\ 0\ \cdots 0)^{\mathrm{T}}$. The transpose of matrix A is A^{T}. The transpose of vector x is x^{T}. I_m denotes the identity matrix in $\mathbb{R}^{m\times m}$. $\mathbb{N}$ denotes the index set $\{1, 2, \cdots, N\}$. For matrix A and B, we let $A \otimes B$ denote their Kronecker product. The gradient and Hessian of function f are ∇f and H, respectively. $\|x\|_p$ denotes the p-norm of the vector x.

Usually, an undirected graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a set of vertices $\mathcal{V} = \{1, 2, \cdots, N\}$ and an edge set $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$. If $i, j \in \mathcal{V}$, and $(i, j) \in \mathcal{E}$, then we say that j is a neighbor of i. The neighbors of vertex i are given by $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. The graph $\mathcal{G}$ is connected, if there has a sequence of distinct vertices such that consecutive vertices are adjacent. The weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is denoted as $a_{ii} = 0$, $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$, otherwise $a_{ij} = 0$. By arbitrarily assigning an orientation for the edges in $\mathcal{G}$. Using $D = [d_{ik}] \in \mathbb{R}^{N\times|\mathcal{E}|}$ represent the incidence matrix associated with the graph $\mathcal{G}$, where $d_{ik} = -1$ if the edge e_k leaves node i, $d_{ik} = 1$ if it enters node i, and $d_{ik} = 0$ otherwise. The degree matrix of $\mathcal{G}$ is $\varLambda = \mathrm{diag}\{d_1, d_2, \cdots, d_N\} \in \mathbb{R}^{N\times N}$, where $d_i = \sum\limits_{j=1, j\neq i}^{N} a_{ij}$ for $i \in \mathbb{N}$. The Laplacian of $\mathcal{G}$ is denoted by

$$L = \varLambda - A.$$

As we know that the Laplacian matrix L is symmetric positive semi-definite and $L = DD^{\mathrm{T}}$. If we denote the eigenvalues of Laplacian L associated with $\mathcal{G}$ with N agents by $\lambda_1(L), \lambda_2(L), \cdots, \lambda_N(L)$, then the following result is well-known[24]

$$\lambda_1(L) = 0 \leqslant \lambda_2(L) \leqslant \cdots \leqslant \lambda_N(L).$$

Lemma 1 The graph $\mathcal{G}$ is connected if and only if $\lambda_1(L) = 0$ and $\mathbf{1}_N = (1\ 1 \cdots 1)^{\mathrm{T}}$ is its eigenvector, and $\lambda_2(L) > 0$.

Lemma 2 The second smallest eigenvalue

$\lambda_2(L)$ of the Laplacian matrix L associated with the undirected connected graph $\mathcal{G}$ satisfies $\lambda_2(L) = \min\limits_{x^{\mathrm{T}}\mathbf{1}_N=0} \dfrac{x^{\mathrm{T}}Lx}{x^{\mathrm{T}}x}$.

Lemma 3[25] Let $f(x):\mathbb{R}^m \to \mathbb{R}$ be a continuously differentiable convex function. $f(x)$ is minimized if and only if $\nabla f = 0$.

3 Time varying convex optimization with flocking behavior

In this paper, the dynamics of all the considered agents can be expressed by the second-order form as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \end{cases} i \in \mathbb{N}, \tag{1}$$

where $x_i(t) \in \mathbb{R}^m$ is the position, $v_i(t) \in \mathbb{R}^m$ is the velocity, and $u_i(t) \in \mathbb{R}^m$ is the control input of agent i. Because $x_i(t)$, $v_i(t)$ and $u_i(t)$ are functions of time, we can rewrite them as x_i, v_i and u_i.

In this paper, the sum of local functions

$$f(v,t) = \sum_{i=1}^{N} f_i(v,t), \tag{2}$$

where $f_i(v,t):\mathbb{R}^m \times \mathbb{R}^+ \to \mathbb{R}$ is a time varying function. Agent i only knows its individual cost $f_i(v,t)$.

Our objective is to devise u_i for (1) using its own cost function and the information gathered from its neighbors such that all agents track the optimal state v^*, and the agents maintain connectivity while avoiding inter-agent collision. Where v^* satisfying

$$v^*(t) = \arg\min_{v\in\mathbb{R}^m} f(v,t). \tag{3}$$

The equation defined in (3) is equivalent to

$$\min_{v_i\in\mathbb{R}^m} \sum_{i=1}^{N} f_i(v_i,t) \ \text{ subject to } \ v_i = v_j. \tag{4}$$

So, the problem is deformed as a minimization problem of the total cost function (2) and a consensus problem. We need the following assumption to deal with the above problem.

Assumption 1 The function $f_i(v,t)$ is convex and continuously twice differentiable with respect to v, with invertible Hessian $H_i(v,t), \forall\, v, t$.

In the following proposed algorithm, each agent can only get its own velocity and its neighbor's velocities. To solve this problem, we present the algorithm.

$$\begin{aligned} u_i(t) = &-\sum_{j\in N_i} \frac{\partial V_{ij}}{\partial x_i} - \alpha \sum_{j\in N_i}(v_i - v_j) - \\ &\beta \sum_{j\in N_i} \operatorname{sgn}(v_i - v_j) + \phi_i, \end{aligned} \tag{5}$$

where

$$\phi_i = -H_i^{-1}(v_i,t)[\tau\nabla f_i(v_i,t) + \frac{\partial \nabla f_i(v_i,t)}{\partial t}],$$

V_{ij} is an artificial potential function of agents i and j to be designed below, α and β are positive coefficients, $\operatorname{sgn}(\cdot)$ is the signum function. It is worth pointing out that ϕ_i depends on only agent i's velocity. We assume that each agent has a limited communication capability, where if $\|x_i - x_j\| < R$, then agent i and j are neighbors. The presented algorithm guarantees preserving connectivity which means that if the initial graph $G(0)$ is connected, $G(t)$ will remain connected for all t. Next, we give the potential function V_{ij} used in the ref [26].

Definition 1[26] The potential function V_{ij} is a differentiable nonnegative function of $\|x_i - x_j\|$ which satisfies the following conditions

1) $V_{ij} = V_{ji}$ has a unique minimum in $\|x_i - x_j\| = d_{ij}$, where d_{ij} is a desired distance between agents i and j and $R > \max\limits_{i,j} d_{ij}$.

2) $V_{ij} \to \infty$ if $\|x_i - x_j\| \to 0$.

3) $\begin{cases} \dfrac{\partial V_{ij}}{\partial(\|x_i - x_j\|)} = 0, & \|x_i(0) - x_j(0)\| \geqslant R, \\ & \|x_i - x_j\| \geqslant R, \\ \dfrac{\partial V_{ij}}{\partial(\|x_i - x_j\|)} \to \infty, & \|x_i(0) - x_j(0)\| < R, \\ & \|x_i - x_j\| \to R. \end{cases}$

Theorem 1 Assume that the initial graph $G(0)$ is connected, The Assumption 1 holds for each agent's cost function $f_i(v_i(t),t)$; and the gradient of the cost functions can be written as $\nabla f_i(v_i,t) = \sigma v_i + g_i(t), \forall\, i \in \mathbb{N}$. If $\alpha > 0$, and $\beta \geqslant \dfrac{2\|\varPhi\|_2}{\sqrt{\lambda_2(L)}}$, for system (1) with the algorithm (5), the agent's velocities track the optimal velocity while preserving connectivity and avoiding collision.

Proof

$$W_1 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} V_{ij} + \frac{1}{2}\sum_{i=1}^{N} v_i^{\mathrm{T}} v_i. \tag{6}$$

Taking time derivative of W_1, we can get

$$\begin{aligned} \dot{W}_1 = &\sum_{i=1}^{N}\sum_{j=1}^{N} v_i^{\mathrm{T}} \frac{\partial V_{ij}}{\partial x_i} + \sum_{i=1}^{N} v_i^{\mathrm{T}} \dot{v}_i = \\ &\sum_{i=1}^{N}\sum_{j=1}^{N} v_i^{\mathrm{T}} \frac{\partial V_{ij}}{\partial x_i} + \sum_{i=1}^{N} v_i^{\mathrm{T}}[-\sum_{j\in N_i}\frac{\partial V_{ij}}{\partial x_i} - \\ &\alpha\sum_{j\in N_i}(v_i - v_j) - \beta\sum_{j\in N_i}\operatorname{sgn}(v_i - v_j) + \phi_i] = \\ &-\beta\sum_{i=1}^{N} v_i^{\mathrm{T}} \sum_{j\in N_i}\operatorname{sgn}(v_i - v_j) - \\ &\alpha\sum_{i=1}^{N} v_i^{\mathrm{T}} \sum_{j\in N_i}(v_i - v_j) + \sum_{i=1}^{N} v_i^{\mathrm{T}}\phi_i. \end{aligned} \tag{7}$$

Because $a_{ij} = a_{ji} = 1$, if $(i,j) \in \mathcal{E}$, we have

$$\begin{aligned} &\sum_{i=1}^{N} v_i^{\mathrm{T}} \sum_{j\in N_i}\operatorname{sgn}(v_i - v_j) = \\ &\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} v_i^{\mathrm{T}} \operatorname{sgn}(v_i - v_j) = \end{aligned}$$

$$\frac{1}{2}[\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}v_i^{\mathrm{T}}\mathrm{sgn}(v_i-v_j)+\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ji}v_j^{\mathrm{T}}\mathrm{sgn}(v_j-v_i)]=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(v_i-v_j)^{\mathrm{T}}\mathrm{sgn}(v_i-v_j). \tag{8}$$

Thus the equation (7) can be rewritten by

$$\dot{W}_1=-\frac{\beta}{2}\sum_{i=1}^{N}\sum_{j\in N_i}(v_i-v_j)^{\mathrm{T}}\mathrm{sgn}(v_i-v_j)-\alpha\sum_{i=1}^{N}v_i^{\mathrm{T}}\sum_{j\in N_i}(v_i-v_j)+\sum_{i=1}^{N}v_i^{\mathrm{T}}\phi_i. \tag{9}$$

If we let $V=(v_1^{\mathrm{T}}\ v_2^{\mathrm{T}}\cdots v_N^{\mathrm{T}})^{\mathrm{T}}$, $\varPhi=(\phi_1^{\mathrm{T}}\ \phi_2^{\mathrm{T}}\cdots\phi_N^{\mathrm{T}})^{\mathrm{T}}$, then the above equation can be rewritten as

$$\begin{aligned}\dot{W}_1=&-\frac{\beta}{2}\|D^{\mathrm{T}}V\|_1-V^{\mathrm{T}}(L\otimes I_m)V+V\varPhi\leqslant\\&-V^{\mathrm{T}}(L\otimes I_m)V-\frac{\beta}{2}\sqrt{V^{\mathrm{T}}(DD^{\mathrm{T}}\otimes I_m)V}+\\&\|V\|_2\|\varPhi\|_2\leqslant\\&-V^{\mathrm{T}}(L\otimes I_m)V-\frac{\beta}{2}\sqrt{\lambda_2(L)}\|V\|_2+\\&\|V\|_2\|\varPhi\|_2=-V^{\mathrm{T}}(L\otimes I_m)V-\\&(\frac{\beta}{2}\sqrt{\lambda_2(L)}-\|\varPhi\|_2)\|V\|_2\leqslant 0.\end{aligned} \tag{10}$$

If $\beta\geqslant\dfrac{2\|\varPhi\|_2}{\sqrt{\lambda_2(L)}}$, then $\dot{W}_1$ is negative semi-definite, by positive semi-definiteness of the Laplacian matrix L. So, we have $W_1\geqslant 0$ and $\dot{W}_1\leqslant 0$, which implies that $W_1(t)\leqslant W_1(0)<\infty$. Moreover, since V_{ij} is bounded, based on Definition 1, it is guaranteed that there will not be inter-agent collision and the connectivity is maintained.

On the other hand, define the level set $\varOmega=\{(x_i-x_j,v_i)|W_1\leqslant c,c>0\}$ which is bounded and closed, so compact. By LaSalle's invariance principle, each solution beginning from $\varOmega$ will converge to the largest invariant set $\{(x_i-x_j,v_i)|\dot{W}_1=0\}$. This occurs only when $v_1=\cdots=v_N$. Which implies that the velocities of all agents in the system (1) asymptotically become the same.

Moreover, in the steady state, becuase $v_i-v_j=0$ for $i,j=1,\cdots,N$, we can have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|x_i-x_j\|^2=2(x_i-x_j)^{\mathrm{T}}(v_i-v_j)=0, \tag{11}$$

and so the distances between agents are unchanged.

In what follows, we devote to find the relation between the agent's velocities and the optimal velocity. Consider the following Lyapunov function

$$W_2=\frac{1}{2}[\sum_{i=1}^{N}\nabla f_i(v_i,t)]^{\mathrm{T}}[\sum_{i=1}^{N}\nabla f_i(v_i,t)], \tag{12}$$

$$\dot{W}_2=[\sum_{i=1}^{N}\nabla f_i(v_i,t)]^{\mathrm{T}}[\sum_{i=1}^{N}H_i(v_i,t)\dot{v}_i+\sum_{i=1}^{N}\frac{\partial}{\partial t}\nabla f_i(v_i,t)]. \tag{13}$$

Because $\nabla f_i(v_i,t)=\sigma v_i+g_i(t),\forall\ i\in I$, we have $H_i(v_i,t)=H_j(v_j,t)$. So, we can get

$$\dot{W}_2=[\sum_{i=1}^{N}\nabla f_i(v_i,t)]^{\mathrm{T}}[H_j(v_j,t)\sum_{i=1}^{N}\dot{v}_i+\sum_{i=1}^{N}\frac{\partial}{\partial t}\nabla f_i(v_i,t)].$$

By summing both sides of the closed-loop system (1) with controller (5) for $j=1,2,\cdots,N$, we can get $\sum\limits_{i=1}^{N}\dot{v}_i=\sum\limits_{i=1}^{N}\phi_i$. Therefor, we have

$$\begin{aligned}\dot{W}_2=&[\sum_{i=1}^{N}\nabla f_i(v_i,t)]^{\mathrm{T}}[H_j(v_j,t)\sum_{i=1}^{N}\phi_i+\sum_{i=1}^{N}\frac{\partial}{\partial t}\nabla f_i(v_i,t)]=\\&-\tau[\sum_{i=1}^{N}\nabla f_i(v_i,t)]^{\mathrm{T}}[\sum_{i=1}^{N}\nabla f_i(v_i,t)].\end{aligned}$$

So, $\dot{W}_2<0$ for $\sum\limits_{i=1}^{N}\nabla f_i(v_i,t)\neq 0$. This guarantees that $\sum\limits_{i=1}^{N}\nabla f_i(v_i,t)$ will asymptotically converge to zero. So, under the assumption that $f_i(v_i,t)$ is convex and applying Lemma 2, we know that as $t\to\infty$, $\sum\limits_{i=1}^{N}f_i(v_i,t)$ will be minimized with $v_i=v_j$, $\forall\ i,j\in\mathbb{N}$.

Remark 1 If $\nabla f_i(v_i,t)$, $\dfrac{\partial}{\partial t}\nabla f_i(v_i,t)$ and $H_i^{-1}(v_i,t)$ are bounded, then $\|\phi_i\|_2$ is bounded. So, the condition $\beta\geqslant\dfrac{2\|\varPhi\|_2}{\sqrt{\lambda_2(L)}}$ of Theorem 1 can be satisfied if $\|\phi_i\|_2$ has a bound.

Remark 2 The research of this paper can be extended to Euler-Lagrangian (EL) systems. That is to say, after the case of the second order system (1) can be solved, each EL agent can track its own virtual system in system (1) to achieve the time-varying optimization. In this way, only local tracking is added for the case of EL system.

Remark 3 In comparison with the mentioned references in the Introduction, the control algorithm (5) enables all agents to implement flocking. The agent's velocity can track the optimal velocity minimizing the sum of time varying cost functions. Moreover, the algorithm is converge faster than the algorithms that only use gradients.

4 Numerical simulations and application

In this section, we present a simulation example to illustrate the theoretical results presented in this paper. We considered 20 agents in 2D plane. We assumed that link range $R = 5$, which means that two agents are neighbors if their this distance is less than R. The agent's task is to have their velocities minimize the total cost function $\sum_{i=1}^{20} f_i(v_i(t), t)$ where $v_i(t) = (v_{x_i}(t)\ v_{y_i}(t))^{\mathrm{T}}$ is the coordinate of agent i in 2D plane.

In the illustration, the second-order dynamic system (1) employed by the control algorithm (5) to minimize the total cost function where the local cost functions are given by

$$f_i(v_i(t), t) = (v_{x_i}(t) - i\sin t)^2 + (v_{y_i}(t) - i\cos t)^2.$$

For the above local cost functions, Assumption 1 and the conditions for agents' cost function in Remark 1 hold and the gradient of the cost functions can be rewritten as $\nabla f_i(x_i; t) = x_i + g_i(t)$. To guarantee the collision avoidance and connectivity maintenance, the potential function partial derivatives is chosen as Eqs. (36) and (37) in [26], where $d_{ij} = 0.5, \forall\, i, j$.

Choosing the coefficients in algorithm (5) as $\alpha = 1, \beta = 8$ and $\tau = 1$. Fig.1 shows the final desired the optimal velocity. Fig.2 gives the final steady state configuration and the final velocity of the agent group. Fig.3 plots the velocity error between the agents and the optimal velocity, and it can be seen that the agent's velocities can track the optimal velocity in deed.

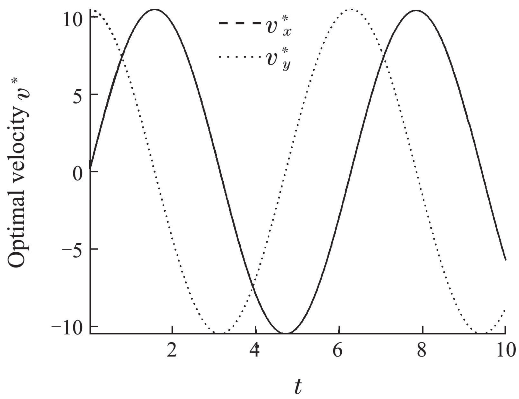

Fig. 1 The optimal velocity of $\sum_{i=1}^{20} f_i(v_i(t), t)$

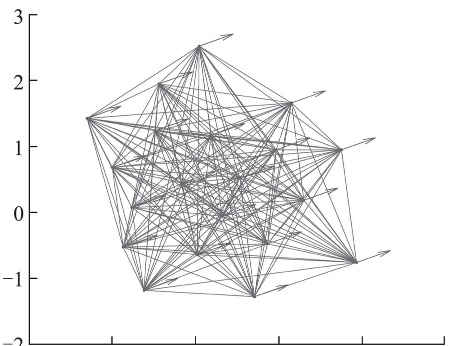

Fig. 2 Final configuration of agents

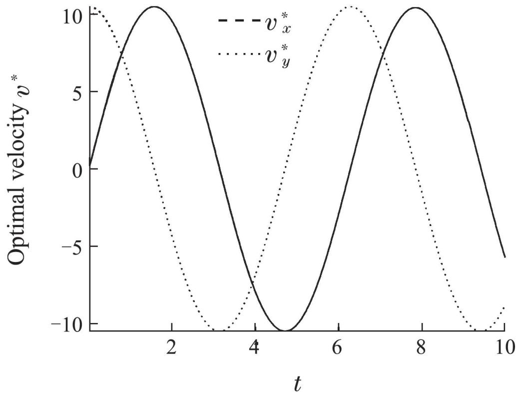

Fig. 3 Errors between agents and the optimal velocity

5 Conclusion

In this paper, we studied optimization problem for continuous-time multi-agent systems with flocking behavior. Multi-agents with second-order dynamics are considered. Each agent is equipped with a time-varying cost function which is known only to an individual agent. The agent's task is to make multi-agent's velocities minimize the sum of local functions by local interaction, while avoiding collision and preserving connectivity. A distributed algorithm with flocking behavior is presented, in which each agent depends only on its own velocity and neighbor's velocities. It is indicated that the velocities of the agents track the optimal velocity. The connectivity of the agents can be maintained and collision between agents is avoided. Moreover, a simulation is included to illustrate the results.

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