文章编号: 1000−8152(2010)02−0205−06

执行器饱和T-S模糊系统的鲁棒耗散容错控制

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摘要: 研究了一类执行器饱和状态变时滞T-S模糊系统的鲁棒容错控制问题. 通过时滞相关Lyapunov函数和对状态的椭球域约束, 基于线性矩阵不等式技术, 提出了非线性系统稳定的不变集条件和模糊鲁棒耗散容错控制器存在的充分条件. 控制方案的设计结果不仅为执行器饱和状态变时滞T-S模糊系统的无源控制和H_∞鲁棒控制建立了统一框架, 而且保证了闭环控制系统对执行器故障的稳定性和容错性. 最后以时滞倒车系统的控制仿真验证了方法的有效性.

关键词: 执行器饱和; T-S模糊系统; 时滞; 耗散控制; 容错控制

中图分类号: TP273 **文献标识码**: A

Robust dissipative tolerant-control of Takagi-Sugeno fuzzy systems with actuator saturation

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Abstract: The problem of robust tolerant-control for a class of Takagi-Sugeno(T-S) fuzzy systems with time-varying state delay and actuator saturation is investigated. By defining a class of delay-dependent Lyapunov function and a given ellipsoid with state constraint, the set of invariance conditions and sufficient conditions for the existence of the robust dissipative tolerant controllers are derived in terms of LMIs. The design results of the control scheme not only provide a unified framework for H-infinity control and passive control, but also ensure the closed-loop systems to be stable and reliable when the actuator faults occur. Simulations of the control of a time-delay truck-trailer system show the effectiveness of the proposed method.

Key words: actuator saturation; T-S fuzzy systems;time-delay; dissipative control; tolerant control

1 引言(Introduction)

在多数实际系统中, 往往都会遇到执行器饱和问题, 这类执行器饱和系统通常呈现为复杂非线性, 并同时存在着由模型时变、信号延迟和元器件磨损老化所造成的严重不确定性和故障隐患. 因此, 研究执行器饱和非线性系统的鲁棒容错控制问题具有重要的理论和应用价值.

近年来, 基于无源化和耗散系统理论的非线性鲁棒控制方法成为了新的研究热点. 动态耗散系统最早由Willems于1972年提出[1]. 耗散系统理论是无源理论、界实引理、卡尔曼–雅柯鲍维奇引理以及圆判据定理的广义化. 基于耗散理论的控制综合可以在系统增益和相位信息之间做出较好的折中, 所得结果保守性较小. 但由于耗散控制的研究成果主要是针对线性系统[2~5], 因此其适用范围十分有限. 考虑到T-S模糊模型可以以任意精度逼近闭集空间上的连续函数, 进而为非线性系统的模糊鲁棒耗散控制问题研究建立了基础[6,7]. 目前, 基于T-S模糊模型的非线性系统的耗散容错控制方法研究尚不多见, 一般也并未考虑执行器所固有的饱和特性影响.

本文针对一类执行器饱和的时滞非线性系统, 基于时滞相关Lyapunov函数和椭球域定义, 给出了系统稳定的不变集条件和鲁棒耗散容错控制器的设计方法. 所得结果为执行器饱和时滞T-S模糊系统的无源控制和H_∞鲁棒控制建立了统一框架, 并保证了闭环控制系统对执行器故障的容错性. 最后以倒车系统的控制仿真验证了方法的有效性.

2 问题描述(Problem statement)

考虑如下一类执行器饱和变时滞T-S模糊系统:

收稿日期: 2009−06−24; 收修改稿日期: 2009−09−20.
基金项目: 国家自然科学基金资助项目(60674092); 航空科学基金资助项目(05E52031); 江南大学创新团队发展计划资助项目.

Rule i : if $z_1(t)$ is F_1^i and $\cdots$ $z_v(t)$ is F_v^i then

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A_i\boldsymbol{x}(t) + A_{\mathrm{d}i}\boldsymbol{x}(t-\tau(t)) + \\ &\quad B_i\sigma(\boldsymbol{u}(t)) + E_i\boldsymbol{w}(t),\\ \boldsymbol{z}(t) &= C_i\boldsymbol{x}(t) + C_{1i}\boldsymbol{x}(t-\tau(t)) + \\ &\quad B_{1i}\sigma(\boldsymbol{u}(t)) + E_{1i}\boldsymbol{w}(t).\end{aligned} \tag{1}$$

式中: $i = 1,2,\cdots,q$, q为模糊规则数; $\boldsymbol{z}(t) = [z_1(t)\ \cdots\ z_v(t)]^{\mathrm{T}} \in \mathbb{R}^v$为模糊推理前件, F_j^i为模糊集合. $\boldsymbol{x}(t)\in\mathbb{R}^n$表示系统的状态向量; $\boldsymbol{u}(t)\in\mathbb{R}^m$表示系统的控制输入向量, $\sigma(\boldsymbol{u}(t)) = [\sigma(u_1(t))\ \sigma(u_2(t))\cdots\sigma(u_{\mathrm{m}}(t))]^{\mathrm{T}}$表示具有饱和特性的执行器输入, 且$\sigma(u_i(t)) = \mathrm{sgn}(u_i(t))\min\{u_{\max,i}, |u_i(t)|\}$, 其中: $u_{\max,i}$表示第i个执行器的饱和值, $i = 1,2,\cdots,m$; $\boldsymbol{w}(t)\in\mathbb{R}^r$表示属于$\mathrm{L}_2[0,\infty)$的扰动向量. $A_i, A_{\mathrm{d}i}, B_i, B_{1i}, C_i, C_{1i}, E_i$和$E_{1i}$是维数适当的实矩阵. $\tau(t)$表示时变滞后时间, 且$0\leqslant\tau(t)\leqslant\tau_{\mathrm{m}}<\infty, \dot{\tau}(t)\leqslant\tau_{\mathrm{d}}$.

令$\bar{S} = \mathrm{diag}\{u_{\max,i}\}$, 并采用单点模糊化, 乘积推理和中心平均清晰化方法, 系统(1)可表示为

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{q}\mu_i(\boldsymbol{z}(t))\{&A_i\boldsymbol{x}(t) + A_{\mathrm{d}i}\boldsymbol{x}(t-\tau(t)) + \\ &\hat{B}_i\hat{\sigma}(\hat{\boldsymbol{u}}(t)) + E_i\boldsymbol{w}(t)\}.\end{aligned} \tag{2}$$

其中: 上式的$\hat{\sigma}(\hat{u}_i(t)) = \mathrm{sgn}(\hat{u}_i(t))\min\{1,|\hat{u}_i(t)|\}$, $\hat{\boldsymbol{u}}(t) = \bar{S}^{-1}\boldsymbol{u}(t)$, $\hat{B}_i = B_i\bar{S}$. 而且

$$\mu_i(\boldsymbol{z}(t)) = \omega_i(\boldsymbol{z}(t))/\sum_{i=1}^{q}\omega_i(\boldsymbol{z}(t)),$$
$$\omega_i(\boldsymbol{z}(t)) = \prod_{j=1}^{v}F_j^i(z_j(t)).$$

而$F_j^i(z_j(t))$表示$z_j(t)$属于模糊集合F_j^i的隶属度, $\omega_i(z(t))$表示第i条规则的权重. 显然

$$\omega_i(\boldsymbol{z}(t))\geqslant 0,\ \sum_{i=1}^{q}\omega_i(\boldsymbol{z}(t))\geqslant 0,\ i = 1,2,\cdots,q,$$

并且

$$\mu_i(\boldsymbol{z}(t))\geqslant 0,\ \sum_{i=1}^{q}\mu_i(\boldsymbol{z}(t)) = 1,\ i = 1,2,\cdots,q.$$

考虑一类常见的执行器结构性失效故障, 有$\sigma_{\mathrm{g}}(\boldsymbol{u}(t)) = G\sigma(\boldsymbol{u}(t))$, 其中: $\sigma_{\mathrm{g}}(\boldsymbol{u}(t))$表示故障情形下的执行器输入, 而$G = \mathrm{diag}\{g_1, g_2,\cdots,g_{\mathrm{m}}\}$表示执行器的故障阵. $0\leqslant g_{lj}\leqslant g_j\leqslant g_{hj}$, $g_{hj}\geqslant 1$, $j = 1,2,\cdots m$. 若令

$$\begin{aligned}&G_0 = \mathrm{diag}\{g_{01}, g_{02},\cdots g_{0m}\},\\ &C = \mathrm{diag}\{c_1, c_2,\cdots c_{\mathrm{m}}\},\\ &L = \mathrm{diag}\{l_1, l_2,\cdots l_{\mathrm{m}}\},\\ &|L| = \mathrm{diag}\{|l_1|, |l_2|,\cdots |l_{\mathrm{m}}|\},\end{aligned}$$

其中:

$$\begin{aligned}&g_{0j} = (g_{hj}+g_{lj})/2,\\ &c_j = (g_{hj}-g_{lj})/(g_{hj}+g_{lj}),\\ &l_j = (g_j - g_{0j})/g_{0j},\end{aligned}$$

则

$$G = G_0(I+L),\ |L|\leqslant C\leqslant I. \tag{3}$$

直接针对执行器故障系统, 设计反馈控制器

Rule i : if $z_1(t)$ is F_1^i and $\cdots$ $z_v(t)$ is F_v^i, then

$$\hat{\sigma}_{\mathrm{g}}(\hat{\boldsymbol{u}}(t)) = G\hat{\sigma}(\hat{\boldsymbol{u}}(t)) = G\hat{\sigma}(\hat{K}_i\boldsymbol{x}(t)). \tag{4}$$

其中: $\hat{K}_i\in\mathbb{R}^{m\times n}$为待定设计的局部增益矩阵, 且$K_i = \bar{S}\hat{K}_i$, 进一步可得清晰化后的闭环系统

$$\begin{cases}\dot{\boldsymbol{x}}(t) = \\ \displaystyle\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(\boldsymbol{z}(t))\mu_j(z(t))\{A_i\boldsymbol{x}(t) + E_i\boldsymbol{w}(t) + \\ \hat{B}_iG\hat{\sigma}(\hat{K}_j\boldsymbol{x}(t)) + A_{\mathrm{d}i}\boldsymbol{x}(t-\tau(t))\},\\ \boldsymbol{z}(t) = \\ \displaystyle\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(\boldsymbol{z}(t))\mu_j(\boldsymbol{z}(t))\{C_i\boldsymbol{x}(t) + E_{1i}\boldsymbol{w}(t) + \\ \hat{B}_{1i}G\hat{\sigma}(\hat{K}_j\boldsymbol{x}(t)) + C_{1i}\boldsymbol{x}(t-\tau(t))\}.\end{cases} \tag{5}$$

针对系统(5), 定义如下形式的二次能量函数[5]:

$$\begin{aligned}&\vartheta(\boldsymbol{w}(t),\boldsymbol{z}(t),T) = \\ &\langle\boldsymbol{z}(t), Q\boldsymbol{z}(t)\rangle_T + 2\langle\boldsymbol{z}(t), S\boldsymbol{w}(t)\rangle_T + \\ &\langle\boldsymbol{w}(t), R\boldsymbol{w}(t)\rangle_T.\end{aligned} \tag{6}$$

其中: $\langle\boldsymbol{x}(t), H\boldsymbol{y}(t)\rangle_T = \int_0^T\boldsymbol{x}^{\mathrm{T}}(t)H\boldsymbol{y}(t)\mathrm{d}t$, Q和R为给定的对称矩阵, S为给定的适维矩阵.

定义 若对任何$T>0$, 存在常数$\alpha>0$, 使得$\vartheta(\boldsymbol{z}(t),\boldsymbol{w}(t),T)+\eta(0)\geqslant\alpha\langle\boldsymbol{w}(t),\boldsymbol{w}(t)\rangle_T$成立, 则称系统(5)严格$(Q,S,R)$鲁棒耗散, 其中$\eta(0)\geqslant 0$.

由于系统(5)中的控制阵$\bar{K}(\boldsymbol{z}(t)) = \sum_{j=1}^{q}\mu_j(\boldsymbol{z}(t))\hat{K}_j$具有饱和特性, 其隶属于多面线性域

$$\begin{aligned}&\chi(\bar{K}(\boldsymbol{z}(t))) = \\ &\left\{\boldsymbol{x}\in\mathbb{R}^n : \left|\bar{k}_i(\boldsymbol{z}(t))\boldsymbol{x}\right|\leqslant 1,\ i = 1,\ 2,\cdots,m\right\}\end{aligned}$$

内, 其中$\bar{k}_i(\boldsymbol{z}(t))$表示控制阵$\bar{K}(\boldsymbol{z}(t))$的第$i$行[8]. 不妨设$\varXi$是对角元素为1或0的$m\times m$维矩阵集合, 集合元素为矩阵$D_i$, $i\in[1,2^m]$. 显然, 若$D_i\in\varXi$, 则$D_i^- = I - D_i\in\varXi$.

引理[9] 对于维数适当的执行器饱和反馈阵

$\bar{K}(\boldsymbol{z}(t))$和$\bar{H}(\boldsymbol{z}(t))$, 若$\forall \boldsymbol{x}(t)\in\chi(\bar{H}(\boldsymbol{z}(t)))$, 则

$$\begin{aligned}&\sigma(\bar{K}(\boldsymbol{z}(t))\boldsymbol{x}(t))\in\\&\mathrm{co}\left\{\left[D_i\bar{K}(\boldsymbol{z}(t))+D_i^-\bar{H}(\boldsymbol{z}(t))\right]\boldsymbol{x}(t)\right\},\end{aligned}$$

其中: $0\leqslant\eta_i(t)\leqslant 1$, $\sum\limits_{i=1}^{2^m}\eta_i(t)=1$, co为凸壳.

因此, 根据引理进一步可得系统的闭环系统为

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\\ \sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}\sum\limits_{l=1}^{2^m}\mu_i(\boldsymbol{z}(t))\mu_j(\boldsymbol{z}(t))\eta_l(t)\{A_{ijl}\boldsymbol{x}(t)+\\ A_{\mathrm{d}i}\boldsymbol{x}(t-\tau(t))+E_i\boldsymbol{w}(t)\},\\ \boldsymbol{z}(t)=\\ \sum\limits_{i=1}^{q}\sum\limits_{j=1}^{q}\sum\limits_{l=1}^{2^m}\mu_i(\boldsymbol{z}(t))\mu_j(\boldsymbol{z}(t))\eta_l(t)\{C_{ijl}\boldsymbol{x}(t)+\\ C_{1i}\boldsymbol{x}(t-\tau(t))+E_{1i}\boldsymbol{w}(t)\}.\end{cases}\tag{7}$$

其中: 系数矩阵$A_{ijl}=A_i+\hat{B}_iG[D_l\hat{K}_j+D_l^-\hat{H}_j]$, $C_{ijl}=C_i+\hat{B}_{1i}G[D_l\hat{K}_j+D_l^-\hat{H}_j]$.

3 主要结果(Main results)

定理 1　考虑闭环系统(7), 若存在矩阵$\tilde{K}_j,\tilde{H}_j$, $j=1,2,\cdots,q$, 对称正定矩阵P^{-1}, $\tilde{S}_1$和$\tilde{S}_2$使得$\varOmega(P,\rho)\subset\chi(\bar{H}(\boldsymbol{z}(t))$, 并满足如下矩阵不等式:

$$\begin{cases}\varOmega_{ijl}=\\ \begin{bmatrix}\varGamma & * & *\\ P^{-1}A_{\mathrm{d}i}^{\mathrm{T}}+\tilde{S}_2 & \varUpsilon & *\\ \varLambda & A_{\mathrm{d}i}P^{-1} & \tau_{\mathrm{m}}^2\tilde{S}_2-2P^{-1}\end{bmatrix}<0,\\ i,j\in[1,q],\ l\in[1,2^m].\end{cases}\tag{8}$$

其中:

$$\begin{aligned}&\varUpsilon=-(1-\tau_{\mathrm{d}})\tilde{S}_1-\tilde{S}_2,\\&\varGamma=P^{-1}A_i^{\mathrm{T}}+A_iP^{-1}+\hat{B}_i(D_l\tilde{K}_j+D_l^-\tilde{H}_j)+\\&\quad\tilde{S}_1+(D_l\tilde{K}_j+D_l^-\tilde{H}_j)^{\mathrm{T}}\hat{B}_i^{\mathrm{T}}-\tilde{S}_2,\\&\varLambda=\hat{B}_i(D_l\tilde{K}_j+D_l^-\tilde{H}_j)+A_iP^{-1},\\&\tilde{S}_1=P^{-1}S_1P^{-1},\ \tilde{S}_2=P^{-1}S_2P^{-1},\\&\tilde{K}_j=\hat{K}_jP^{-1},\ \tilde{H}_j=\hat{H}_jP^{-1},\end{aligned}$$

则椭球域 $\varOmega(P,\rho)$ 是不变集, 且当 $\boldsymbol{w}(t)=0$ 时系统(1)在饱和控制器(4)作用下稳定.

证　针对系统(7)构造时滞相关Lyapunov函数

$$\begin{aligned}&V(\boldsymbol{x}(t))=\\&\boldsymbol{x}^{\mathrm{T}}(t)P\boldsymbol{x}(t)+\int_{t-\tau(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(\theta)S_1\boldsymbol{x}(\theta)\mathrm{d}\theta+\\&\tau_{\mathrm{m}}\int_{t-\tau_{\mathrm{m}}}^{t}(\theta-(t-\tau_{\mathrm{m}}))\dot{\boldsymbol{x}}^{\mathrm{T}}(\theta)S_2\dot{\boldsymbol{x}}(\theta)\mathrm{d}\theta.\end{aligned}\tag{9}$$

其中: P, S_1和S_2均为对称正定矩阵, 对式(9)求导, 并由Leibniz-Newton公式

$$\begin{aligned}&-\tau_{\mathrm{m}}\int_{t-\tau_{\mathrm{m}}}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(\theta)S_2\dot{\boldsymbol{x}}(\theta)\mathrm{d}\theta\leqslant\\&-(\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau(t))^{\mathrm{T}}S_2(\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau(t)).\end{aligned}\tag{10}$$

进一步整理后可得

$$\begin{aligned}&\dot{V}(\boldsymbol{x}(t)\leqslant\\&\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(\boldsymbol{z}(t))\mu_j(\boldsymbol{z}(t))\sum_{l=1}^{2^m}\eta_l(t)\{\bar{\boldsymbol{x}}^{\mathrm{T}}(t)\ \varOmega_{ijl}\bar{\boldsymbol{x}}(t)\}.\end{aligned}\tag{11}$$

其中:

$$\bar{\boldsymbol{x}}^{\mathrm{T}}(t)=[\boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\,\dot{\boldsymbol{x}}^{\mathrm{T}}(t)],$$

$$\begin{cases}\varOmega_{ijl}=\\ \begin{bmatrix}\varGamma_1 & * & *\\ A_{\mathrm{d}i}^{\mathrm{T}}P+S_2 & -(1-\tau_{\mathrm{d}})S_1-S_2 & *\\ \varLambda_1 & PA_{\mathrm{d}i} & \tau_{\mathrm{m}}^2S_2-2P\end{bmatrix},\\ i,j\in[1,q],\ l\in[1,2^m],\end{cases}\tag{12}$$

$$\begin{aligned}&\varGamma_1=A_i^{\mathrm{T}}P+PA_i+P\hat{B}_i(D_l\hat{K}_j+D_l^-\hat{H}_j)+\\&\quad S_1-S_2+(D_l\hat{K}_j+D_l^-\hat{H}_j)^{\mathrm{T}}\hat{B}_i^{\mathrm{T}}P,\\&\varLambda_1=PA_i+P\hat{B}_i(D_l\hat{K}_j+D_l^-\hat{H}_j).\end{aligned}$$

在式(12)左右两边同时乘以$\mathrm{diag}\{P^{-1},P^{-1},P^{-1}\}$后即可得定理1中的式(8)形式, 因此, 若$\varOmega_{ijl}<0$, 则$\dot{V}<0$, 从而可知椭球域$\varOmega(P,\rho)$是不变集, 即执行器饱和控制器(4)作用下的闭环系统(7)在吸引域$\varOmega(P,\rho)$内稳定.　　证毕.

定理 2　考虑闭环系统(7), 对于给定矩阵S, 对称矩阵Q,R和$T>0$, 若存在矩阵$\tilde{K}_j$, $\tilde{H}_j$, $j=1,2,\cdots,q$, 对称正定矩阵P^{-1}, $\tilde{S}_1$, $\tilde{S}_2$和实数α, 使得$\varOmega(P,\rho)\subset\chi(\bar{H}(z(t))$, 并满足如下矩阵不等式:

$$\begin{cases}\varTheta_{ijl}=\\ \begin{bmatrix}\varTheta_{11} & * & * & * & *\\ P^{-1}A_{\mathrm{d}i}^{\mathrm{T}}+\tilde{S}_2 & \varTheta_{22} & * & * & *\\ \varTheta_{31} & A_{\mathrm{d}i}P^{-1} & \varTheta_{33} & * & *\\ \varTheta_{41} & \varTheta_{42} & E_i^{\mathrm{T}} & \varTheta_{44} & *\\ \varTheta_{51} & C_{1i}P^{-1} & 0 & E_{1i} & -T\end{bmatrix}<0,\\ i,j\in[1,q],\ l\in[1,2^m].\end{cases}\tag{13}$$

其中:

$$\begin{aligned}&\varTheta_{11}=\\&P^{-1}A_i^{\mathrm{T}}+A_iP^{-1}+\hat{B}_i(D_l\tilde{K}_j+D_l^-\tilde{H}_j)+\\&\tilde{S}_1+(D_l\tilde{K}_j+D_l^-\tilde{H}_j)^{\mathrm{T}}\hat{B}_i^{\mathrm{T}}-\tilde{S}_2,\\&\varTheta_{22}=-(1-\tau_{\mathrm{d}})\tilde{S}_1-\tilde{S}_2,\end{aligned}$$

$\varTheta_{31} = A_iP^{-1} + \hat{B}_i(D_l\tilde{K}_j + D_l^-\tilde{H}_j),$
$\varTheta_{33} = \tau_{\rm m}^2\tilde{S}_2 - 2P^{-1},$
$\varTheta_{41} = E_i^{\rm T} - S^{\rm T}C_iP^{-1} - S\hat{B}_{1i}[D_l\tilde{K}_j + D_l^-\tilde{H}_j],$
$\varTheta_{42} = -S^{\rm T}C_{1i}P^{-1},$
$\varTheta_{44} = -(R-\alpha I) - S^{\rm T}E_{1i} - E_{1i}^{\rm T}S,$
$\varTheta_{51} = C_iP^{-1} + \hat{B}_{1i}[D_l\tilde{K}_j + D_l^-\tilde{H}_j],$
$-Q < T^{-1},\ \tilde{S}_1 = P^{-1}S_1P^{-1},$
$\tilde{S}_2 = P^{-1}S_2P^{-1},\ \tilde{K}_j = \hat{K}_jP^{-1},\ \tilde{H}_j = \hat{H}_jP^{-1},$

则椭球域$\varOmega(P,\rho)$是不变集, 并对所有$w(t)\in{\rm L}_2$, 闭环系统(7)严格(Q,S,R)鲁棒耗散.

证 由定理1可知, 当系统(7)的$\boldsymbol{w}(t)\neq 0$时:

$$\dot{V}(\boldsymbol{x}(t) - \boldsymbol{z}^{\rm T}(t)Qz(t) - 2\boldsymbol{w}^{\rm T}(t)S\boldsymbol{z}(t) - \boldsymbol{w}^{\rm T}(t)(R-\alpha I)\boldsymbol{w}(t) \leqslant \widehat{\boldsymbol{x}}^{\rm T}(t)\varTheta_{ijl}\widehat{\boldsymbol{x}}(t).$$

其中:

$$\widehat{\boldsymbol{x}}^{\rm T}(t) = [\boldsymbol{x}^{\rm T}(t)\ \boldsymbol{x}^{\rm T}(t-\tau(t))\ \dot{\boldsymbol{x}}^{\rm T}(t)\ \boldsymbol{w}^{\rm T}(t)],$$

$$\varTheta_{ijl}=\begin{bmatrix}\varGamma_1 - C_{ijl}^{\rm T}QC_{ijl} & * & * & *\\ \varDelta_{21} & \varDelta_{22} & * & *\\ PA_{ijl} & PA_{\rm di} & \tau_{\rm m}^2S_2-2P & *\\ \varDelta_{41} & \varDelta_{42} & E_i^{\rm T}P & \varDelta_{44}\end{bmatrix},$$

$i,j\in[1,q],\ l\in[1,2^m].$

其中:

$$\varGamma_1 = P\hat{B}_i(D_l\hat{K}_j + D_l^-\hat{H}_j) + (D_l\hat{K}_j + D_l^-\hat{H}_j)^{\rm T}\hat{B}_i^{\rm T}P + A_i^{\rm T}P + PA_i + S_1 - S_2,$$
$$\varDelta_{21} = A_{\rm di}^{\rm T}P + S_2 - C_{1i}^{\rm T}QC_{ijl},$$
$$\varDelta_{22} = -(1-\tau_{\rm d})S_1 - S_2 - C_{1i}^{\rm T}QC_{1i},$$
$$\varDelta_{41} = E_i^{\rm T}P - S^{\rm T}C_{ijl} - E_{1i}^{\rm T}QC_{ijl},$$
$$\varDelta_{42} = -S^{\rm T}C_{1i} - E_{1i}^{\rm T}QC_{1i},$$
$$\varDelta_{44} = -(R-\alpha I) - S^{\rm T}E_{1i} - E_{1i}^{\rm T}S - E_{1i}^{\rm T}QE_{1i}.$$

由于$-Q<T^{-1}$, 且$T^{-1}>0$, 则

$$\varTheta_{ijl}\leqslant\begin{bmatrix}\varGamma_1 & * & * & * & *\\ A_{\rm di}^{\rm T}P+S_2 & \bar{\varDelta}_{22} & * & * & *\\ PA_{ijl} & PA_{\rm di} & \bar{\varDelta}_{33} & * & *\\ E_i^{\rm T}P - S^{\rm T}C_{ijl} & -S^{\rm T}C_{1i} & E_i^{\rm T}P & \bar{\varDelta}_{44} & *\\ C_{ijl} & C_{1i} & 0 & E_{1i} & -T\end{bmatrix}. \tag{14}$$

其中:

$$\bar{\varDelta}_{22} = -(1-\tau_{\rm d})S_1 - S_2,\ \bar{\varDelta}_{33} = \tau_{\rm m}^2S_2 - 2P,$$
$$\bar{\varDelta}_{44} = -(R-\alpha I) - S^{\rm T}E_{1i} - E_{1i}^{\rm T}S,$$

在式(14)左右两边同时乘以${\rm diag}\{P^{-1},P^{-1},P^{-1},I,I\}$, 并将

$$A_{ijl} = \hat{B}_i[D_l\hat{K}_j + D_l^-\hat{H}_j] + A_i$$

和

$$C_{ijl} = C_i + \hat{B}_{1i}[D_l\hat{K}_j + D_l^-\hat{H}_j]$$

代入上式, 即可得式(13)形式, 则

$$\dot{V}(\boldsymbol{x}(t)) - \boldsymbol{z}^{\rm T}(t)Q\boldsymbol{z}(t) - 2\boldsymbol{w}^{\rm T}(t)S\boldsymbol{z}(t) - \boldsymbol{w}^{\rm T}(t)(R-\alpha I)\boldsymbol{w}(t) < 0. \tag{15}$$

将式(15)左右两边从0到∞积分可得

$$\vartheta(\boldsymbol{z}(t),\boldsymbol{w}(t),\infty) + V(\boldsymbol{x}(0)) - V(\boldsymbol{x}(\infty)) > \alpha\left\langle\boldsymbol{w}(t),\boldsymbol{w}(t)\right\rangle_\infty,$$

从而可得$\vartheta(\boldsymbol{z}(t),\boldsymbol{w}(t),\infty)+\eta(0)\geqslant\alpha\left\langle\boldsymbol{w}(t),\boldsymbol{w}(t)\right\rangle_\infty$, 即系统(7)严格$(Q,S,R)$鲁棒耗散. 证毕.

定理 3 考虑执行器故障的系统(7), 对于给定矩阵S, 对称矩阵Q、R和$T>0$, 若存在矩阵$\tilde{K}_j$, $\tilde{H}_j$, $j=1,2,\cdots,q$, 对称正定矩阵P^{-1}, $\tilde{S}_1$, $\tilde{S}_2$, R_1, R_2和实数$\alpha>0$, 使得$\varOmega(P,\rho)\subset\chi(\bar{H}(z(t)))$, 并满足

$$\begin{cases}\varPsi_{ijk}=\begin{bmatrix}\varPsi_{11} & * & * & * & * & * & *\\ P^{-1}A_{\rm di}^{\rm T}+\tilde{S}_2 & -(1-\tau_{\rm d})\tilde{S}_1-\tilde{S}_2 & * & * & * & * & *\\ \varPsi_{31} & A_{\rm di}P^{-1} & \varPsi_{33} & * & * & * & *\\ \varPsi_{41} & \varPsi_{42} & E_i^{\rm T} & \varPsi_{44} & * & * & *\\ \varPsi_{51} & C_{1i}P^{-1} & 0 & \varPsi_{54} & \varPsi_{55} & * & *\\ \varPsi_{61} & 0 & 0 & 0 & 0 & -R_1 & *\\ \varPsi_{71} & 0 & 0 & 0 & 0 & 0 & -R_2\end{bmatrix}<0,\\ i,j\in[1,q],\ l\in[1,2^m].\end{cases} \tag{16}$$

其中:

$$\varPsi_{11} = P^{-1}A_i^{\rm T} + A_iP^{-1} + \hat{B}_iG_0(D_l\tilde{K}_j + D_l^-\tilde{H}_j) + (D_l\tilde{K}_j + D_l^-\tilde{H}_j)^{\rm T}G_0^{\rm T}\hat{B}_i^{\rm T} + \hat{B}_iG_0R_1G_0^{\rm T}\hat{B}_i^{\rm T} + \tilde{S}_1 - \tilde{S}_2,$$
$$\varPsi_{31} = A_iP^{-1} + \hat{B}_iG_0(D_l\tilde{K}_j + D_l^-\tilde{H}_j) + \hat{B}_iG_0R_1G_0^{\rm T}\hat{B}_i^{\rm T},$$
$$\varPsi_{33} = \tau_{\rm m}^2\tilde{S}_2 + \hat{B}_iG_0R_1G_0^{\rm T}\hat{B}_i^{\rm T} - 2P^{-1},$$

$$\Psi_{41}=E_i^{\mathrm{T}}-S^{\mathrm{T}}C_iP^{-1}-S^{\mathrm{T}}\hat{B}_{1i}G_0[D_l\tilde{K}_j+D_l^-\tilde{H}_j],$$
$$\Psi_{42}=-S^{\mathrm{T}}C_{1i}P^{-1},$$
$$\Psi_{44}=-S^{\mathrm{T}}E_{1i}-E_{1i}^{\mathrm{T}}S-(R-\alpha I)+S\hat{B}_{1i}G_0R_2G_0^{\mathrm{T}}\hat{B}_{1i}^{\mathrm{T}}S^{\mathrm{T}},$$
$$\Psi_{51}=\hat{B}_{1i}G_0[D_l\tilde{K}_j+D_l^-\tilde{H}_j]+C_iP^{-1},$$
$$\Psi_{54}=-2\hat{B}_{1i}G_0R_2G_0^{\mathrm{T}}\hat{B}_{1i}^{\mathrm{T}}S^{\mathrm{T}}+E_{1i},$$
$$\Psi_{55}=-T+\hat{B}_{1i}G_0R_2G_0^{\mathrm{T}}\hat{B}_{1i}^{\mathrm{T}},$$
$$\Psi_{61}=\Psi_{71}=D_l\tilde{K}_j+D_l^-\tilde{H}_j,$$
$$-Q<T^{-1},\ \tilde{S}_1=P^{-1}S_1P^{-1},$$
$$\tilde{S}_2=P^{-1}S_2P^{-1},\ \tilde{K}_j=\hat{K}_jP^{-1},$$
$$\tilde{H}_j=\hat{H}_jP^{-1},$$

则椭球域$\Omega(P,\rho)$是不变集, 并对所有$w(t)\in \mathrm{L}_2$, 故障系统(7)在控制器(4)作用下严格(Q,S,R)鲁棒耗散.

证　当闭环系统(7)的执行器发生结构性故障时,
$$A_{ijl}=A_i+\hat{B}_iG[D_l\hat{K}_j+D_l^-\hat{H}_j],$$
$$C_{ijl}=C_i+\hat{B}_{1i}G[D_l\hat{K}_j+D_l^-\hat{H}_j],$$
在代入式(13)后, 并根据矩阵性质和Schur补变换可得式(16)成立.

从定理 3 可知, 由于执行器饱和闭环系统要求$\Omega(P,\rho)\subset\chi(\bar{H}(\boldsymbol{z}(t)))$, 即$\forall\boldsymbol{x}(t)\in\Omega(P,\rho)$, $\left|\bar{h}_l(\boldsymbol{z}(t))\boldsymbol{x}(t)\right|\leqslant 1,\ l\in[1,m]$, 从文献[10]可知
$$\bar{h}_l(\boldsymbol{z}(t))P^{-1}\bar{h}_l^{\mathrm{T}}(\boldsymbol{z}(t))\leqslant\rho^{-1},\ l\in[1,m].\quad(17)$$
由于$\mu_i(\boldsymbol{z}(t))\geqslant 0,\sum_{i=1}^{q}\mu_i(\boldsymbol{z}(t))=1$, 因此不妨令
$$\hat{h}_l^jP^{-1}(\hat{h}_l^j)^{\mathrm{T}}\leqslant\rho^{-1},\ l\in[1,m],\ j\in[1,q].\quad(18)$$
其中$\hat{h}_l^j$为第j个局部增益矩阵$\hat{H}_j$的第l行. 则
$$\begin{bmatrix}-\rho^{-1} & \tilde{h}_l^j\\ * & -P^{-1}\end{bmatrix}\leqslant 0,\ l\in[1,2^m],\ j\in[1,q],\quad(19)$$
而$\tilde{h}_l^j$为矩阵$\tilde{H}_j=\hat{H}_jP^{-1}$的第$l$行.

4　仿真结果(Simulation results)

仿真以3阶模型倒车系统为被控对象, 选取$\theta(t)$在“0”与“$\pm\pi$”两个情形下进行T-S模糊建模, 可得

Rule 1 : if θ is about 0, then
$$\dot{\boldsymbol{x}}(t)=A_1\boldsymbol{x}(t)+A_{\mathrm{d}1}\boldsymbol{x}(t-\tau(t))+B_1\sigma(u(t))+E_1w(t),$$
$$\boldsymbol{z}(t)=C_1\boldsymbol{x}(t)+C_{11}\boldsymbol{x}(t-\tau(t))+B_{11}\sigma(u(t))+E_{11}w(t),$$

Rule 2 : if θ is about $\pm\ \pi$, then
$$\dot{\boldsymbol{x}}(t)=A_2\boldsymbol{x}(t)+A_{\mathrm{d}2}\boldsymbol{x}(t-\tau(t))+B_2\sigma(u(t))+E_2w(t),$$
$$\boldsymbol{z}(t)=C_2\boldsymbol{x}(t)+C_{12}\boldsymbol{x}(t-\tau(t))+B_{12}\sigma(u(t))+E_{12}w(t),$$

$$A_1=\begin{bmatrix}-a\dfrac{V\bar{t}}{Lt_0} & 0 & 0\\ a\dfrac{V\bar{t}}{Lt_0} & 0 & 0\\ a\dfrac{V^2\bar{t}^2}{2Lt_0} & \dfrac{V\bar{t}}{t_0} & 0\end{bmatrix},\ B_1=B_2=\begin{bmatrix}\dfrac{V\bar{t}}{Lt_0}\\ 0\\ 0\end{bmatrix},$$
$$A_2=\begin{bmatrix}a\dfrac{V\bar{t}}{Lt_0} & 0 & 0\\ a\dfrac{V\bar{t}}{Lt_0} & 0 & 0\\ ad\dfrac{V^2\bar{t}^2}{2Lt_0} & d\dfrac{V\bar{t}}{t_0} & 0\end{bmatrix},\ E_1=E_2=\begin{bmatrix}1\\0\\1\end{bmatrix},$$
$$A_{\mathrm{d}2}=\begin{bmatrix}-(1-a)\dfrac{Vt}{Lt_0} & 0 & 0\\ (1-a)\dfrac{Vt}{Lt_0} & 0 & 0\\ (1-a)d\dfrac{V^2t^2}{2Lt_0} & 0 & 0\end{bmatrix},\ B_{11}=B_{12}=\begin{bmatrix}1\\1\end{bmatrix},$$
$$A_{\mathrm{d}1}=\begin{bmatrix}-(1-a)\dfrac{V\bar{t}}{Lt_0} & 0 & 0\\ (1-a)\dfrac{V\bar{t}}{Lt_0} & 0 & 0\\ (1-a)\dfrac{V^2\bar{t}^2}{2Lt_0} & 0 & 0\end{bmatrix},\ E_{11}=E_{12}=\begin{bmatrix}0.8\\1\end{bmatrix},$$
$$C_1=C_2=\begin{bmatrix}1&0&2\\0&1&1\end{bmatrix},\ C_{11}=C_{12}=\begin{bmatrix}2&1&0\\1&0&0\end{bmatrix},$$

其中: $\boldsymbol{x}(t)=[x_1(t)\,x_2(t)\,x_3(t)]^{\mathrm{T}}$, $x_1(t)$表示卡车和拖车间的角度差, $x_2(t)$表示拖车角度, $x_3(t)$表示拖车后端垂直位置, $\sigma(u(t))$表示饱和执行器, 饱和值$u_{\max}=4$. 卡车长$l=2.8$ m, 拖车长$L=5.5$ m, 倒车速度$V=-1.0$ m/s, $\bar{t}=2.0,\ t_0=0.5,\ a\in[0,1)$为滞后项系数. 在仿真中取$a=0.5$, 滞后时间$\tau(t)=2+0.2\sin\ t,\ d=0.01/\pi$. 而模糊隶属函数
$$\mu_1(\theta(t))=(1-(1/(1+\mathrm{e}^{-3\theta(t)-0.5\pi})))(1-(1/(1+\mathrm{e}^{-3\theta(t)+0.5\pi}))),$$
$$\mu_2(\theta(t))=1-\mu_1(\theta(t)),$$
因此, 当执行器因结构故障失效75 %时, 此时执行器

的饱和值减小为$u_{\max}=1$. 若取$S=[1\ \ 1]^{\mathrm{T}}$, $Q=\mathrm{diag}\{-1,-2\}$, $R=0.6$, $\alpha=0.1$, 并同时选择$T=\mathrm{diag}\{0.5,0.4\}$, 则进一步根据定理3可得容错控制器增益

$$K_1=[9.8547\ \ -6.5218\ \ 0.6536],$$
$$K_2=[10.3730\ \ -6.6993\ \ 0.6572].$$

仿真时的初始位置$\boldsymbol{x}^{\mathrm{T}}(0)=[0.5\pi\ \ 0.75\pi\ \ 1]^{\mathrm{T}}$, $w(t)$是白噪声信号. 图1和图2分别给出了故障倒车系统在控制器(4)作用下的状态响应和饱和执行器$\sigma(u(t))$的信号曲线. 可以看出, 在故障情形下本文方法保证了系统的稳定, 并较好地抑制了扰动影响.

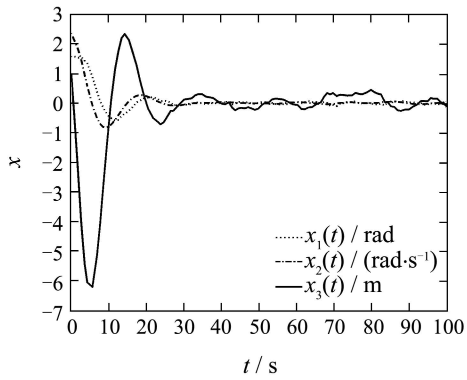

图 1 执行器故障时闭环系统的状态响应

Fig. 1 Responses of the closed-loop system when the actuator failures occur

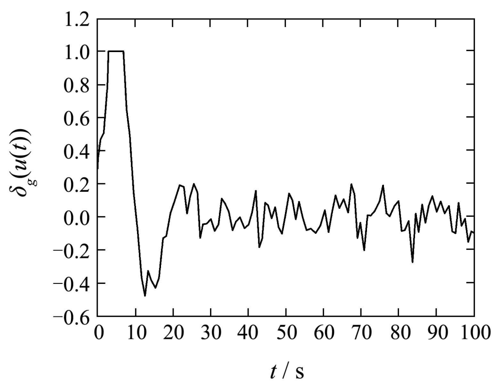

图 2 饱和执行器的信号曲线

Fig. 2 Signal curve of the saturated actuator

5 结语(Conclusion)

本文研究了一类T-S模糊系统在一定稳态及动态性能指标下的鲁棒可靠输出反馈控制问题, 提出了传感器或执行器在正常及发生结构性故障情形下的控制器设计方法, 并利用投射引理降低设计的保守性. 仿真结果表明了本方法的可行性.

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