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Adaptive neural-fuzzy control of triple inverted pendulum

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Abstract: In the triple inverted pendulum(TIP) system, adaptive neural-fuzzy inference system(ANFIS) approach is atilized to combine fuzzy logic with Neural-Network, according to the input/output data, so that ANFIS automatically adjusts fuzzy rules and membership functions based on state synthesis to fit sampling data. The simulation results show that the designed ANFIS controller is feasible. Compared with LQR control, triple inverted pendulum based on ANFIS control has better dynamics performance and anti-interference capability.

Keywords: triple inverted pendulum; adaptive neural-fuzzy inference; state synthesis; LQR

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三级倒立摆的自适应神经模糊控制

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摘要: 在三级倒立摆(TIP)系统中, 应用神经网络与模糊控制相结合的自适应神经模糊推理系统(adaptive neural-fuzzy inference system), 根据样本数据调整隶属函数和控制规则参数, 使得训练后ANFIS控制器很好地模拟期望的输入输出数据. 仿真结果表明所设计的ANFIS控制器对三级倒立摆系统的稳定控制是可行的. 与LQR控制相比, 基于ANFIS控制的倒立摆系统具有良好的动态性能和抗干扰性能.

关键词: 三级倒立摆; 自适应神经模糊推理系统; 状态合成; LQR

1 Introduction

Inverted pendulum system is a non-linear, strong coupling, multivariable and absolutely unstable system. Its stability control can effectively reflect many key issues of the automatic control, such as stability, non-linearity, servo problems, tracking problems, and so on. In addition, in practical application, many objects have similar motion characteristics with inverted pendulum, such as attitude control of the satellite, movement of the robot's joint, etc. Therefore, there is theoretical and practical significance in the study of inverted pendulum.

For conventional fuzzy controllers, the arbitrariness in setting membership function parameters and the difficulty in rules proposition are difficult issues in fuzzy controller design, especially for a multivariable system. Adaptive neural-fuzzy inference system(ANFIS) is used in this paper, which is functionally equivalent to the adaptive network of fuzzy inference system. It not only has the characteristics of fuzzy control that does not require accurate model and strong robust features, but also has the characteristics of self-learning of neural network. ANFIS controller is designed according to this theory and applied to triple inverted pendulum (TIP) control.

2 Model of triple inverted pendulum

Fig.1 shows the illustration of the TIP system[1]. Here, M is the mass of the cart; m_1 represents the mass of the first pendulum; m_2 represents the mass of the second pendulum; m_3 represents the mass of the third pendulum; M_1, M_2 represent the mass between two pendulums. l_1, l_2, l_3 denote the distance between the pivot and the center of mass of respective links.

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$J_i = \dfrac{1}{3}m_i l_i^2, i = 1, 2, 3$ represents the moment of inertia of the pendulum. g denotes the acceleration of gravity.

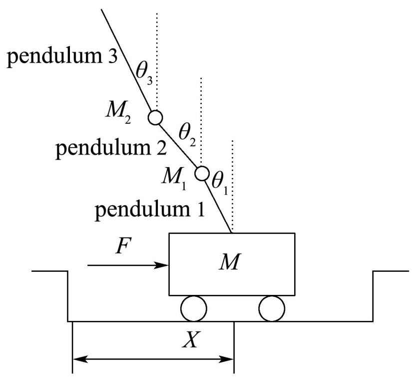


Fig. 1 Triple inverted pendulum system

For the TIP system, we select generalized coordinates as: $x,\ \theta_1,\ \theta_2,\ \theta_3$. We neglect the frictions between pendula and frictions between cart and pendulum. According to the Euler-Lagrangian equation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\partial L}{\partial \dot{\theta}_i}\Big) - \frac{\partial L}{\partial \theta_i} = f_i,\ i = 1, 2, 3,$$

we can obtain the nonlinear equations as follows:

$$\begin{aligned}&-k_1\ddot{x}l_1\cos\theta_1 + 2k_2l_1^2\ddot{\theta}_1 + 2k_4l_1l_2\ddot{\theta}_2\cos(\theta_1 - \\&\theta_2) + 2m_3l_1l_3\ddot{\theta}_3\cos(\theta_1-\theta_3) + \\&2k_4l_1l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) + \\&2m_3l_1l_3\dot{\theta}_3^2\sin(\theta_1-\theta_3) - k_1gl_1\sin\theta_1 = 0,\end{aligned} \tag{1}$$

$$\begin{aligned}&-k_4\ddot{x}l_2\cos\theta_2 + 2k_4l_1l_2\ddot{\theta}_1\ \cos(\theta_1-\theta_2) + \\&2k_3l_2^2\ddot{\theta}_2\ + 2m_3l_2l_3\ddot{\theta}_3\cos(\theta_2-\theta_3) - \\&2k_4l_1l_2\dot{\theta}_1^2\sin(\theta_1-\theta_2) + \\&2m_3l_2l_3\dot{\theta}_3^2\sin(\theta_2-\theta_3) - k_4gl_2\sin\theta_2 = 0,\end{aligned} \tag{2}$$

$$\begin{aligned}&2m_3l_1l_3\ddot{\theta}_1\cos(\theta_1-\theta_3) + 2m_3l_2l_3\ddot{\theta}_2\cos(\theta_2 - \\&\theta_3) - 2m_3l_1l_3\dot{\theta}_1^2\sin(\theta_1-\theta_3) - \\&2m_3l_2l_3\dot{\theta}_2^2\sin(\theta_2-\theta_3) - \\&m_3\ddot{x}l_3\cos\theta_3 + \frac{4}{3}m_3l_3^2\ddot{\theta}_3 - \\&m_3gl_3\sin\theta_3 = 0.\end{aligned} \tag{3}$$

Where:

$$\begin{aligned}k_1 &= m_1 + 2m_2 + 2m_3 + 2M_1 + 2M_2,\\ k_2 &= \frac{2}{3}m_1 + 2m_2 + 2m_3 + 2M_1 + 2M_2,\\ k_3 &= \frac{2}{3}m_2 + 2m_3 + 2M_2,\\ k_4 &= m_2 + 2m_3 + 2M_2.\end{aligned}$$

Considering the pendulum would swing in the neighborhood of the desired equilibrium, the above equations can be linearized by Taylor's Formula. Then the state equation of TIP can be achieved:

Where:

$$\begin{bmatrix}\dot{x}\\ \dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \ddot{x}\\ \ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix} = \begin{bmatrix}0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0\\ 0&k_{12}&k_{13}&k_{14}&0&0&0&0\\ 0&k_{22}&k_{23}&k_{24}&0&0&0&0\\ 0&k_{32}&k_{33}&k_{34}&0&0&0&0\end{bmatrix}\begin{bmatrix}x\\ \theta_1\\ \theta_2\\ \theta_3\\ \dot{x}\\ \dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\\1\\k_{19}\\k_{29}\\k_{39}\end{bmatrix}u.$$

Where:

$$\begin{aligned}k_{12} &= \frac{-2k_4l_2k_{22} - 2m_3l_3k_{32} + k_1g}{2k_2l_1},\\ k_{13} &= \frac{-2k_4l_2k_{23} - 2m_3l_3k_{33}}{2k_2l_1},\\ k_{14} &= \frac{-2k_4l_2k_{24} - 2m_3l_3k_{34}}{2k_2l_1},\\ k_{19} &= \frac{-2k_4l_2k_{29} - 2m_3l_3k_{39} + k_1}{2k_2l_1}.\end{aligned}$$

$$\begin{aligned}k_{22} &= \frac{(\frac{4}{3}k_4 - 2m_3)k_1k_4g}{(2k_4^2 - 2k_2k_3)(2m_3l_2 - \frac{4}{3}k_4l_2) - (2k_4 - 2k_2)(2k_3m_3l_2 - 2k_4m_3l_2)},\\ k_{23} &= \frac{(2m_3 - \frac{4}{3}k_4)k_2k_4g + (2k_4 - 2k_2)k_4m_3g}{(2k_4^2 - 2k_2k_3)(2m_3l_2 - \frac{4}{3}k_4l_2) - (2k_4 - 2k_2)(2k_3m_3l_2 - 2k_4m_3l_2)},\\ k_{24} &= \frac{-(2k_4 - 2k_2)k_4m_3g}{(2k_4^2 - 2k_2k_3)(2m_3l_2 - \frac{4}{3}k_4l_2) - (2k_4 - 2k_2)(2k_3m_3l_2 - 2k_4m_3l_2)},\end{aligned}$$

$$k_{29} = \frac{(k_2k_4 - k_1k_4)(2m_3 - \frac{4}{3}k_4)}{(2k_4^2 - 2k_2k_3)(2m_3l_2 - \frac{4}{3}k_4l_2) - (2k_4 - 2k_2)(2k_3m_3l_2 - 2k_4m_3l_2)},$$

$$k_{32} = \frac{(2k_4 - 2k_3)k_1k_4g}{(2k_4 - 2k_2)(2k_3m_3l_3 - 2m_3k_4l_3) - (2k_4^2 - 2k_2k_3)(2m_3l_3 - \frac{4}{3}k_4l_3)},$$

$$k_{33} = \frac{k_2k_4g(2k_3 - 2k_4) + (2k_4^2 - 2k_2k_3)k_4g}{(2k_4 - 2k_2)(2k_3m_3l_3 - 2m_3k_4l_3) - (2k_4^2 - 2k_2k_3)(2m_3l_3 - \frac{4}{3}k_4l_3)},$$

$$k_{34} = \frac{-(2k_4^2 - 2k_2k_3)k_4g}{(2k_4 - 2k_2)(2k_3m_3l_3 - 2m_3k_4l_3) - (2k_4^2 - 2k_2k_3)(2m_3l_3 - \frac{4}{3}k_4l_3)},$$

$$k_{39} = \frac{(k_2k_4 - k_1k_4)(2k_3 - 2k_4)}{(2k_4 - 2k_2)(2k_3m_3l_3 - 2m_3k_4l_3) - (2k_4^2 - 2k_2k_3)(2m_3l_3 - \frac{4}{3}k_4l_3)}.$$

3 ANFIS

ANFIS is a fuzzy inference system based on Takagi-Sugeno model, which adopt a structure similar to the neural network[2~6]. It can use the learning mechanism of neural network to adjust fuzzy inference system of membership functions on the basis of input and output sample data automatically, draw rules, and then form an adaptive neural fuzzy controller. For a fuzzy inference system with two inputs(x and y) and one output (z), with one order Sugeno fuzzy model, two fuzzy rules are as follows:
1) If x is A_1 and y is B_1, then $f_1 = p_1x + q_1y + r_1$;
2) If x isA_2 and y is B_2, then $f_2 = p_2x + q_2y + r_2$.
Fig.2 shows the equivalent ANFIS structure of the Sugeno fuzzy model.

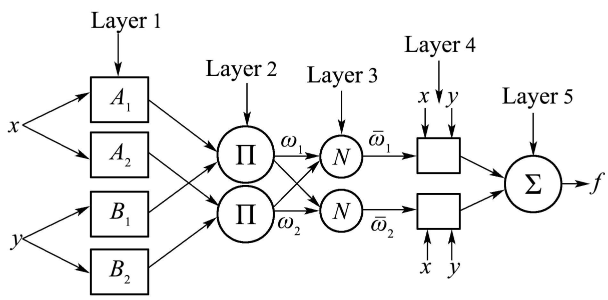


Fig. 2 Equivalent ANFIS structure

Layer 1 The node in this layer is an adaptive node. x (or y) is the input of node i , A (or B) is language of identity (such as "small" or "big") related to the node. In this layer, the parameters which can determine the membership function are the premise parameters:

$$O_{1,i} = \mu_{A_i}(x),\ i = 1, 2,$$

or

$$O_{1,i} = \mu_{B_{i-2}}(x),\ i = 3, 4.$$

Layer 2 In this layer, each node is a symbol $\prod$ to the fixed nodes,whose output is the product of all the input signals:

$$O_{2,i} = \omega_i = \mu_{A_i}(x)\mu_{B_i}(y),\ i = 1, 2.$$

Layer 3 In this layer, each node is symbolized by N and calculates the ratio of the related incentive strength to the total, which is showed as follows. The output of this layer is called the normalization of incentive strength.

$$O_{3,i} = \bar{\omega}_i = \frac{\omega_i}{\omega_1 + \omega_2},\ i = 1, 2.$$

Layer 4 The node in this layer is an adaptive node. The $\bar{\omega}_i$ in the following equation is the normalization of incentive strength from layer 3. $\{p_i, q_i, r_i\}$ is the parameter set of the node. The parameter of this layer is called a conclusion parameter.

$$O_{4,i} = \bar{\omega}_i f_i = \bar{\omega}_i(p_ix + q_iy + r_i).$$

Layer 5 In this layer, each node is symbolized by $\sum$ and calculates the sum of the signals.

$$\mathrm{sum} = O_{5,i} = \sum \bar{\omega}_i f_i = \frac{\sum\limits_i \bar{\omega}_i f_i}{\sum\limits_i \omega_i}.$$

In this way, an adaptive network equal to the Sugeno fuzzy models in function is constructed.

4 Application of ANFIS controller in TIP system

4.1 Reducing the control variable dimension

For a TIP system, there are 8 state variables: x, θ_1, θ_2, θ_3, $\dot{x}$, $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$. The conventional fuzzy control method can cause too many rules[7~13]. In order to design an effective controller, down- dimensional vector processing will be used on the 8 state variables. In this paper, state feedback coefficients based on the optimal control are adopted to form synthesized error and synthesized error rate and fuzzy controller is derived from this which can reduce the dimension of input variables and the number of rules. The 8 state variables can be processed according to the method mentioned above:

$$\begin{aligned} &E = K_1^{\mathrm{T}} X_1 = \\ &[k_1\, k_2\, k_3\, k_4][x\, \theta_1\, \theta_2\, \theta_3]^{\mathrm{T}} = \\ &k_1 x + k_2\theta_1 + k_3\theta_2 + k_4\theta_3, \end{aligned} \tag{4}$$

$$\begin{aligned} &EC = K_2^{\mathrm{T}} X_2 = \\ &[k_5\, k_6\, k_7\, k_8][\dot{x}\, \dot{\theta}_1\, \dot{\theta}_2\, \dot{\theta}_3]^{\mathrm{T}} = \\ &k_5\dot{x} + k_6\dot{\theta}_1 + k_7\dot{\theta}_2 + k_8\dot{\theta}_3. \end{aligned} \tag{5}$$

The initial value of state variable synthesis coefficients ($K^{\mathrm{T}} = [k_1\, k_2\, k_3\, k_4\, k_5\, k_6\, k_7\, k_8]$) synthesis coefficients (K_0) is obtained from the solution to the Riccati equation

$$PA + A^{\mathrm{T}}P - PBR^{-1}B^{\mathrm{T}}P + C^{\mathrm{T}}QC = 0.$$

The solution to the equation is

$$K_0^{\mathrm{T}} = R^{-1}B^{\mathrm{T}}P.$$

K in state variable synthesis is from the adjusted state feedback coefficient of LQR controller. Then the synthesized error of ANFIS is

$$E = -3.16\times x + 82.05\times\theta_1 - 9.15\times\theta_2 - 7.58\times\theta_3.$$

The ratio of the synthesized error is

$$EC = -6.39\times\dot{x} + 6.91\times\dot{\theta}_1 + 4.37\times\dot{\theta}_2 + 4.30\times\dot{\theta}_3.$$

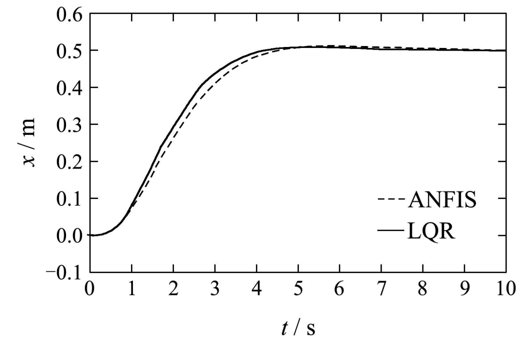
4.2 Structure of ANFIS controller

The ANFIS controller adopts Sugeno, and two inputs are synthesized error E and ratio of the synthesized error EC. The output is u. The range of E and EC is $[-6, 6]$, and that both have 8 fuzzy subsets, $\{$NB NM NS NZ PZ PS PM PB$\}$. There are 8 membership functions that Gauss membership functions are adopted. The range of output of the controller (u) is $[-10, 10]$ and one order linear output is adopted. The input space is divided into 64 intervals, which correspond to 64 fuzzy rules.

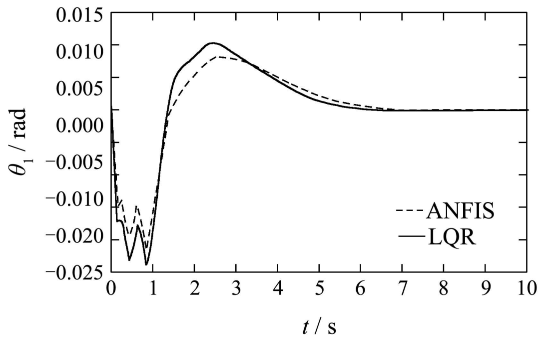
4.3 Training of ANFIS controller

LQR controller is obtained from optimal theory and used in the TIP system to obtain input and output data. The sample time is set as 0.01 s and 1000 sample data are regarded as the training data of the ANFIS controller. The ANFISEDIT function is also used in the paper. Firstly, load the training data and the FIS structure. Secondly, select the study algorithm. There are two study algorithms in ANFISEDIT, backpropa and Hybrid. In the paper, the latter is adopted. At last, set the error tolerance and training epochs. In this algorithm, make sure the initial value of the premise parameters $\{a_i, b_i\}$, adjust the conclusion parameters $\{p_i, q_i, r_i\}$ by least square method and update the premise parameters by backpropa algorithm.

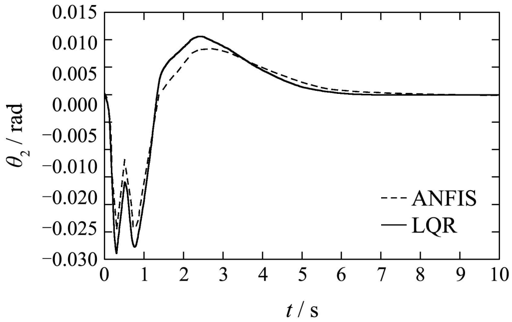
Through the study of training data, the premise parameters and conclusion parameters are adjusted. The training error is only 1.55×10^{-5} after 1000 epochs. The trained fuzzy inference system can analogize the expected input and output data well.



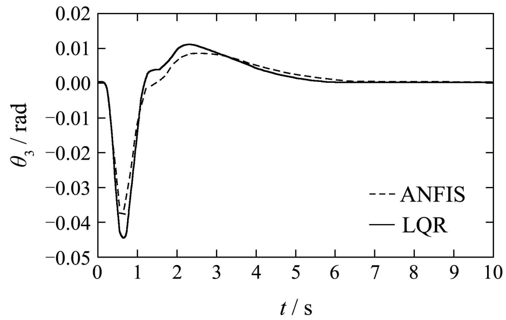
(a) Position



(b) Angle of pendulum 1



(c) Angle of pendulum 2



(d) Angle of pendulum 3

Fig. 3 Simulation results

4.4 Simulation results

Apply the ANFIS controller and the LQR controller to TIP system. The target location is 0.5 and the initial state is $x^{\mathrm{T}} = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$. The simulation results are shown in Fig.3. From the simulation results, the use of the designed ANFIS controller can successfully realize the control of the TIP system. The simulation figures show that the angle is smaller based on the ANFIS controller. The largest angel is not more than $0.6°$. Compared with LQR, overshoot of ANFIS controller is better.

5 Conclusion

Adaptive-neural fuzzy inference system overcomes the human crucial factors in fuzzification and defuzzification and the incompleteness and roughness of fuzzy rules in fuzzy inference systems. It uses the input and output data to adjust membership functions and form fuzzy rules automatically. ANFIS controller is used to realize the control of TIP system. According to the comparison of the simulation curves, the ANFIS controller can improve the dynamic performance of the TIP system in a large scale and it also shows good adaptability of the controlled plant.

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