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网络化神经网络的时滞依赖稳定性判据

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摘要: 本文研究了网络化神经网络的稳定性问题. 首先, 为了利用网络系统的采样特征, 定义了一个新的Lyapunov泛函; 通过分析网络诱导时延和执行周期之间的关系, 采用一个迭代凸组合技术, 得到了一个包含较少保守性的稳定性判据. 然后, 给出一个基于采样数据的神经网络稳定性判据, 减少了计算复杂性. 最后, 通过一个数例, 验证了本文方法的有效性和优越性.

关键词: 神经网络; 采样控制; 稳定性条件

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Delay-dependent stability criteria for network-based neural networks

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Abstract: This paper investigates the problem of stability of network-based neural networks (NNs). To exploit the sampling characteristic of network systems, we define a new type of Lyapunov functional. By analyzing the relation between the network-induced delay and the executive duration, and employing an iterative convex combination technique, we develop a less conservative stability criterion for network-based NNs. To reduce the computational complexity, we also propose a stability criterion for sampled-data-based NNs. An illustrative example is given to show the effectiveness and the advantages of the proposed method.

Key words: neural networks (NNs); sampled-data control; stability criteria

1 Introduction

Neural networks (NNs) have immense potentials of application prospective in a variety of areas, such as signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization. Due to the finite speed of information processing, the existence of time delays frequently causes oscillation, divergence, or instability in NNs. In recent years, the stability problem of delayed neural networks has become a topic of great theoretic and practical importance [1–14].

For NNs with single time-varying delay, there exist many delay-dependent stability results reported in the literature (see [5–6, 15–16] and the references therein). In [6], delay-dependent stability condition was derived by defining a new Lyapunov functional and the obtained condition could include some existing time delay-independent ones. In [15], a less conservative delay-dependent stability criterion for delayed NNs was proposed by using the free-weighting matrix method and considering the useful term when estimating the upper bound of the derivative of Lyapunov functional. And the stability result in [15] was improved in [16], where a convex combination technique [17] was employed.

Recently, a new model for neural networks with two additive time-varying delays had been proposed, and this kind of neural network has a strong application background in remote control and network-based control [18]. In a network-based system, signals transmitted from one point to another may experience two segments of networks, and the resulting time delays have different properties due to variable network transmission conditions. For such network-based NNs, [18] presented a stability criterion by using the free-weighting matrix method. And the stability result in [18] was improved in [19] by remaining some integral terms and using a convex combination technique [17].

It is worth pointing out that the considered network-based NNs were modeled as delayed NNs with two additive time-varying delay components, and the sampling characteristic of network-based systems was ignored. In

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network-based systems, the received signal by receiptor is kept invariable in each executive period, which is realized with a zero-order holder (ZOH). This sampling characteristic shows that network-based NNs belong to a special class of NNs with single time-varying delay, so the stability criteria for network-based NNs are expected to be less conservative than the ones for general delayed NNs, which motivates this study.

In this paper, the problem of stability analysis for NNs with two additive time-varying delay components is investigated. By considering the independence and the variation of two additive time-varying delay components, a new type of Lyapunov functionals is proposed. Combining with a tighter estimation of the derivative of the Lyapunov functional and a reciprocally convex combination technique [20], new delay-dependent stability criteria with less conservatism and less complexity are derived in terms of linear matrix inequalities (LMIs). A numerical example is also given to show the effectiveness and the significant improvement of the proposed method.

Notation: A real symmetric matrix $P > 0(\geqslant 0)$ denotes P being a positive definite (positive semi-definite) matrix, and $A > B(A \geqslant B)$ means $A - B > 0(\geqslant 0)$. I is used to denote an identity matrix with proper dimension. The symmetric terms in a symmetric matrix are denoted by $*$. For a square matrix E, $\mathrm{He}(E)$ is defined as $\mathrm{He}(E) = E + E^{\mathrm{T}}$.

2 Problem formulation

Consider a mode of network-based NN as follows:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(s_k) + u,$$
$$t \in [t_k,\ t_{k+1}),\ k \in \mathbb{N}, \tag{1}$$

where $x(\cdot) = [x_1(\cdot)\ \ \cdots\ \ x_n(\cdot)]^{\mathrm{T}} \in \mathbb{R}^n$ is the neuron state vector, $g(x(\cdot)) = [g_1(x_1(\cdot))\ \ \cdots\ \ g_n(x_n(\cdot))]^{\mathrm{T}} \in \mathbb{R}^n$ denotes the neuron activation function, and $u = [u_1\ \ \cdots\ \ u_n]^{\mathrm{T}} \in \mathbb{R}^n$ is a constant input vector. $C = \mathrm{diag}\{c_1,\ \cdots,\ c_n\}$ with $c_i > 0(i = 1,\ 2,\ \cdots,\ n)$, and $A,\ B$ are the connection weight matrix and the delayed connection weight matrix, respectively. N denotes the set of all nonnegative integers; t_k denotes the instant that the destination neuron receives the k-th signal, which is sampled by the sensor of the source neuron at sampling instant s_k; τ_k stands for the network-inducted delay (causing by transmission delay and discarding the outdated data packets) of the data packet sampled at instant s_k from the sensor to the destination, that is, $\tau_k = t_k - s_k$; The term $t - t_k$ is called as the executive duration in k-th executive period $[t_k,\ t_{k+1})$.

According to [21] and [22], the following assumption is made for network-based NN (1):

Assumption 1 The sampling-delay sequences $\{s_k, \tau_k\}$ $(k \in N)$ satisfy

$$(s_{k+1} - s_k) + \tau_{k+1} \leqslant \eta, \tag{2}$$
$$\tau_{\mathrm{m}} \leqslant \tau_k \leqslant \tau_{\mathrm{M}}, \tag{3}$$

where $\tau_{\mathrm{m}},\ \tau_{\mathrm{M}}$ and η are nonnegative constants.

Remark 1 In the network environment, it is usually assumed that the sensor is clock-driven, while the acceptor and zero-order hold (ZOH) are event-driven. If one packet sampled at the sensor node reaches the destination later than its successors, then it will be dropped and the latest one will be used. This guarantees that both $\{t_k\}$ and $\{s_k\}$ are strictly increasing sequences. In condition (2), η can be used to reflect the allowable bound on the amount of the data dropout and the network-induced delays.

Assume that the activation functions $g_i(\cdot)(i = 1,\ 2,\ \cdots,\ n)$ are bounded and satisfy

$$0 \leqslant \frac{g_i(x) - g_i(y)}{x - y} \leqslant \sigma_i,\ \forall x,\ y \in \mathbb{R},\ x \neq y, \tag{4}$$

where $\sigma_i(i = 1,\ 2,\ \cdots,\ n)$ are some constants.

From the Brouwer's fixed-point theorem, there exists an equilibrium point for (1). Assume that $x^* = [x_1^*\ \ x_2^*\ \ \cdots\ \ x_n^*]^{\mathrm{T}}$ is an equilibrium point of system (1), by choosing the coordinate transformation $z(\cdot) = x(\cdot) - x^*$, system (1) is changed into the following error system

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(s_k)), \tag{5}$$

where $t \in [t_k,\ t_{k+1})$ and $z(\cdot) = [z_1(\cdot)\ \ \cdots\ \ z_n(\cdot)]^{\mathrm{T}}$ is the state vector of the transformed system, $f(z) = [f_1(z_1(\cdot))\ \ \cdots\ \ f_n(z_n(\cdot))]^{\mathrm{T}}$ and $f_i(z_i(\cdot)) = g_i(z_i(\cdot) + x_i^*) - g_i(x_i^*)(i = 1,\ 2,\ \cdots,\ n)$. Then, the functions $f_i(\cdot)(i = 1,\ 2,\ \cdots,\ n)$ satisfy the following condition:

$$0 \leqslant \frac{f_i(z_i)}{z_i} \leqslant \sigma_i,\ f_i(0) = 0,\ \forall z_i \neq 0, \tag{6}$$

which implies that

$$(f_i(z_i(t)) - \sigma_i z_i(t)) \cdot f_i(z_i(t)) \leqslant 0 \tag{7}$$

and

$$(f_i(z_i(s_k)) - \sigma_i z_i(s_k)) \cdot f_i(z_i(s_k)) \leqslant 0 \tag{8}$$

where $t > 0$ and $i = 1,\ 2,\ \cdots,\ n$.

Obviously, network-based NN (5) belongs to a special class of NNs with single delay:

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t - \tau(t))), \tag{9}$$

where $\tau(t) = t - s_k(t \in [t_k,\ t_{k+1}))$ and $\tau(t)$ satisfies $\tau_{\mathrm{m}} \leqslant \tau(t) \leqslant \eta$. For such delayed NNs, there exist many results of stability analysis reported in the literature (see [6, 15–16]).

When $\tau_{\mathrm{M}} = 0$, it is known than $\tau_k = 0$ and

$t_k = s_k$. Thus, network-based NN (5) reduces to

$$\begin{aligned}\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t_k)),\\ t \in [t_k,\ t_{k+1}),\ k \in \mathbb{N},\end{aligned} \tag{10}$$

and Assumption 1 becomes Assumption 2.

Assumption 2 The sampling sequences $t_k (k \in N)$ satisfy

$$0 < t_{k+1} - t_k \leqslant \eta, \tag{11}$$

where η is a positive constant.

In this paper, the stability problems of both network-based and sampled-data-based NNs are studied. The sampling characteristic of networked systems is captured by defining a new type of Lyapunov-Krasovskii functional (LKF), and the coupled relationship of the network-induced delay and the executive duration is clarified, while an iterative convex combination technique is utilized, which leads to a less conservative stability criterion for network-based NNs. That is, a larger allowable upper bound of η can be yielded by the obtained stability criterion. Then, to reduce the computational complexity, a stability criterion for sampled-data-based NNs is also proposed. Finally, an illustrative example is given to show the effectiveness and the improvement of the proposed method.

3 Main results

For network-based NN (5), a Lyapunov-Krasovskii functional can be chosen as

$$V(z(t)) = V_1(z(t)) + V_2(z(t)) + V_3(z(t)) + V_4(z(t)), \tag{12}$$

where

$$\begin{aligned}
V_1(z(t)) &= z^{\mathrm{T}}(t)Pz(t) + 2\sum_{i=1}^{n}\lambda_{1i}\int_0^{z_i(t)} f_i(s)\mathrm{d}s + \\
&\quad 2\sum_{i=1}^{n}\lambda_{2i}\int_0^{z_i(t)}(\sigma_i s - f_i(s))\mathrm{d}s,\\
V_2(z(t)) &= (\eta - (t - s_k))\int_{t_k}^{t}\dot{z}^{\mathrm{T}}(s)R\dot{z}(s)\mathrm{d}s,\\
V_3(z(t)) &= \int_{t-\eta}^{t} z^{\mathrm{T}}(s)Uz(s)\mathrm{d}s,\\
V_4(z(t)) &= \int_{-\eta}^{0}\int_{t+\theta}^{t}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s\mathrm{d}\theta,
\end{aligned}$$

and $P = P^{\mathrm{T}} > 0,\ R = R^{\mathrm{T}} > 0,\ U = U^{\mathrm{T}} \geqslant 0,\ Z = Z^{\mathrm{T}} > 0,\ \varLambda_j = \mathrm{diag}\{\lambda_{j1},\ \lambda_{j2},\ \cdots,\ \lambda_{jn}\} \geqslant 0 (j = 1, 2)$ are to be determined.

Obviously, $V(z(t))$ is discontinuous. However, at any $t > t_0$ except the jumps t_k, $V(z(t))$ is continuous and positive. Note that $V_2(z(t_{k+1}^-)) \geqslant 0$ and $V_2(z(t_{k+1}^+)) = 0$, this shows that $V_2(z(t))$ does not increase along the jumps t_k $(k \in \mathbb{N})$. Therefore, along the jumps t_k, $V(z(t))$ also does not increase.

For convenience, we denote $\varSigma = \mathrm{diag}\{\sigma_1,\ \sigma_2,\ \cdots,\ \sigma_n\}$ and

$$\begin{aligned}
&e_1 = [I\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],\ e_2 = [0\ \ I\ \ 0\ \ 0\ \ 0\ \ 0],\\
&e_3 = [0\ \ 0\ \ I\ \ 0\ \ 0\ \ 0],\ e_4 = [0\ \ 0\ \ 0\ \ I\ \ 0\ \ 0],\\
&e_5 = [0\ \ 0\ \ 0\ \ 0\ \ I\ \ 0],\ e_6 = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ I].
\end{aligned}$$

Now, we give a new stability criterion for the origin of system (5) as follows.

Theorem 1 For given scalars $\tau_{\mathrm{m}},\ \tau_{\mathrm{M}},\ \eta (\tau_{\mathrm{m}} \leqslant \tau_{\mathrm{M}} \leqslant \eta)$, the origin of system (5) under Assumption 1 is globally asymptotically stable if there exist matrices $P = P^{\mathrm{T}} > 0,\ R = R^{\mathrm{T}} > 0,\ Z = Z^{\mathrm{T}} > 0,\ U = U^{\mathrm{T}} \geqslant 0,\ \varLambda_j = \mathrm{diag}\{\lambda_{j1},\ \lambda_{j2},\ \cdots,\ \lambda_{jn}\} \geqslant 0,\ D_j = \mathrm{diag}\{d_{j1},\ d_{j2},\ \cdots,\ d_{jn}\} \geqslant 0,\ j = 1,\ 2$, and matrices $Y,\ T,\ M$ with appropriate dimensions, such that

$$\begin{bmatrix} \varPsi & \mathcal{A}^{\mathrm{T}}Q_1 & (\eta - \tau_{\mathrm{m}})M & \tau_{\mathrm{m}}T\\ * & -Q_1 & 0 & 0\\ * & * & -(\eta - \tau_{\mathrm{m}})Z & 0\\ * & * & * & -\tau_{\mathrm{m}}Z \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \varPsi & \mathcal{A}^{\mathrm{T}}Q_2 & (\eta - \tau_{\mathrm{M}})M & \tau_{\mathrm{M}}T\\ * & -Q_2 & 0 & 0\\ * & * & -(\eta - \tau_{\mathrm{M}})Z & 0\\ * & * & * & -\tau_{\mathrm{M}}Z \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \varPsi & \eta\mathcal{A}^{\mathrm{T}}Z & (\eta - \tau_{\mathrm{m}})Y & \tau_{\mathrm{m}}T\\ * & -\eta Z & 0 & 0\\ * & * & -(\eta - \tau_{\mathrm{m}})(R + Z) & 0\\ * & * & * & -\tau_{\mathrm{m}}Z \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} \varPsi & \eta\mathcal{A}^{\mathrm{T}}Z & (\eta - \tau_{\mathrm{M}})Y & \tau_{\mathrm{M}}T\\ * & -\eta Z & 0 & 0\\ * & * & -(\eta - \tau_{\mathrm{M}})(R + Z) & 0\\ * & * & * & -\tau_{\mathrm{M}}Z \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned}
\varPsi = {} & \mathrm{He}(e_1^{\mathrm{T}}(P + \varSigma\varLambda_2)\mathcal{A} + e_5^{\mathrm{T}}(\varLambda_1 - \varLambda_2)\mathcal{A} + \\
& Y(e_1 - e_2) + T(e_2 - e_3) + M(e_3 - e_4)) + \\
& e_1^{\mathrm{T}}Ue_1 - e_4^{\mathrm{T}}Ue_4 - 2e_5^{\mathrm{T}}D_1e_5 + e_5^{\mathrm{T}}D_1\varSigma e_1 + \\
& e_1^{\mathrm{T}}\varSigma D_1e_5 - 2e_6^{\mathrm{T}}D_2e_6 + e_6^{\mathrm{T}}D_2\varSigma e_3 + \\
& e_3^{\mathrm{T}}\varSigma D_2e_6,\\
Q_1 = {} & \eta Z + (\eta - \tau_{\mathrm{m}})R,\\
Q_2 = {} & \eta Z + (\eta - \tau_{\mathrm{M}})R,\\
\mathcal{A} = {} & \begin{bmatrix} -C & 0 & 0 & 0 & A & B \end{bmatrix}.
\end{aligned}$$

Proof Denote $\zeta(t) = [\zeta_1^{\mathrm{T}}(t)\ \ \zeta_2^{\mathrm{T}}(t)]^{\mathrm{T}}$ with $\zeta_1(t) = [z^{\mathrm{T}}(t)\ \ z^{\mathrm{T}}(t_k)\ \ z^{\mathrm{T}}(s_k)\ \ z^{\mathrm{T}}(t-\eta)]^{\mathrm{T}}$ and $\zeta_2(t) = [f^{\mathrm{T}}(z(t))\ \ f^{\mathrm{T}}(z(s_k))]^{\mathrm{T}}$.

Taking the time derivative of $V_i(z(t)) (i = 1,\ 2,\ 3,\ 4)$ along the trajectory of (5) yields that

$$\dot{V}_1(z(t)) = $$

$$2z^{\mathrm{T}}(t)P\dot{z}(t) + 2\sum_{i=1}^{n}\lambda_{1i}f_i(z_i(t))\dot{z}_i(t) +$$

$$2\sum_{i=1}^{n}\lambda_{2i}(\sigma_i z_i(t)-f_i(z_i(t)))\dot{z}_i(t)=$$
$$2z^{\mathrm{T}}(t)(P+\Sigma\Lambda_2)\dot{z}(t)+$$
$$2f^{\mathrm{T}}(z(t))(\Lambda_1-\Lambda_2)\dot{z}(t)=$$
$$\zeta^{\mathrm{T}}(t)\mathrm{He}(e_1^{\mathrm{T}}(P+\Sigma\Lambda_2)\mathcal{A}+e_5^{\mathrm{T}}(\Lambda_1-\Lambda_2)\mathcal{A})\zeta(t),$$
$$\dot{V}_2(z(t))=(\eta-(t-s_k))\zeta^{\mathrm{T}}(t)\mathcal{A}^{\mathrm{T}}R\mathcal{A}\zeta(t)-\int_{t_k}^{t}\dot{z}^{\mathrm{T}}(s)R\dot{z}(s)\mathrm{d}s,$$
$$\dot{V}_3(z(t))=$$
$$z^{\mathrm{T}}(t)Uz(t)-z^{\mathrm{T}}(t-\eta)Uz(t-\eta)=$$
$$\zeta^{\mathrm{T}}(t)(e_1^{\mathrm{T}}Ue_1-e_4^{\mathrm{T}}Ue_4)\zeta(t),$$
$$\dot{V}_4(z(t))=$$
$$\eta\zeta^{\mathrm{T}}(t)\mathcal{A}^{\mathrm{T}}Z\mathcal{A}\zeta(t)-\int_{t-\eta}^{t}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s.$$

It is known that

$$-\int_{t_k}^{t}\dot{z}^{\mathrm{T}}(s)R\dot{z}(s)\mathrm{d}s-\int_{t-\eta}^{t}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s=$$
$$-\int_{t_k}^{t}\dot{z}^{\mathrm{T}}(s)(R+Z)\dot{z}(s)\mathrm{d}s-\int_{s_k}^{t_k}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s-$$
$$\int_{t-\eta}^{s_k}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s,$$

and

$$-\int_{t_k}^{t}\dot{z}^{\mathrm{T}}(s)(R+Z)\dot{z}(s)\mathrm{d}s\leqslant$$
$$\zeta^{\mathrm{T}}(t)((t-t_k)Y(R+Z)^{-1}Y^{\mathrm{T}}+\mathrm{He}(Y(e_1-e_2)))\zeta(t),$$
$$-\int_{s_k}^{t_k}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s\leqslant$$
$$\zeta^{\mathrm{T}}(t)(\tau_k TZ^{-1}T^{\mathrm{T}}+\mathrm{He}(T(e_2-e_3)))\zeta(t),$$
$$-\int_{t-\eta}^{s_k}\dot{z}^{\mathrm{T}}(s)Z\dot{z}(s)\mathrm{d}s\leqslant$$
$$\zeta^{\mathrm{T}}(t)((\eta-(t-s_k))MZ^{-1}M^{\mathrm{T}}+\mathrm{He}(M(e_3-e_4)))\zeta(t).$$

On the other hand, from (7) and (8), it yields that

$$0\leqslant$$
$$-2f^{\mathrm{T}}(z(t))D_1(f(z(t))-\Sigma z(t))=$$
$$\zeta^{\mathrm{T}}(t)(-2e_5^{\mathrm{T}}D_1e_5+e_5^{\mathrm{T}}D_1\Sigma e_1+e_1^{\mathrm{T}}\Sigma D_1e_5)\zeta(t),$$
$$0\leqslant$$
$$-2f^{\mathrm{T}}(z(s_k))D_2(f(z(s_k)-\Sigma z(s_k))=$$
$$\zeta^{\mathrm{T}}(t)(-2e_6^{\mathrm{T}}D_2e_6+e_6^{\mathrm{T}}D_2\Sigma e_3+e_3^{\mathrm{T}}\Sigma D_2e_6)\zeta(t).$$

So, it gets that

$$\dot{V}(z(t))\leqslant\zeta^{\mathrm{T}}(t)(\Psi_0+(t-s_k)\Psi_1+\tau_k\Psi_2)\zeta(t),\tag{17}$$

where

$$\Psi_0=$$
$$\mathrm{He}(e_1^{\mathrm{T}}(P+\Sigma\Lambda_2)\mathcal{A}+e_5^{\mathrm{T}}(\Lambda_1-\Lambda_2)\mathcal{A}+$$
$$Y(e_1-e_2)+T(e_2-e_3)+M(e_3-e_4))+$$
$$e_1^{\mathrm{T}}Ue_1-e_4^{\mathrm{T}}Ue_4-2e_5^{\mathrm{T}}D_1e_5+e_5^{\mathrm{T}}D_1\Sigma e_1+$$
$$e_1^{\mathrm{T}}\Sigma D_1e_5-2e_6^{\mathrm{T}}D_2e_6+e_6^{\mathrm{T}}D_2\Sigma e_2+$$
$$e_2^{\mathrm{T}}\Sigma D_2e_6+\eta\mathcal{A}^{\mathrm{T}}(R+Z)\mathcal{A}+\eta MZ^{-1}M^{\mathrm{T}},$$
$$\Psi_1=Y(R+Z)^{-1}Y^{\mathrm{T}}-\mathcal{A}^{\mathrm{T}}R\mathcal{A}-MZ^{-1}M^{\mathrm{T}},$$
$$\Psi_2=-Y(R+Z)^{-1}Y^{\mathrm{T}}+TZ^{-1}T^{\mathrm{T}}.$$

From (2) in Assumption 1, it is known that

$$\tau_k\leqslant t-s_k\leqslant t_{k+1}-s_k\leqslant\eta,\tag{18}$$

which shows that $t-s_k$ varies within $[\tau_k,\ \eta]$. By using the convex combination technique given in [17], it gets that $\Psi_0+(t-s_k)\Psi_1+\tau_k\Psi_2<0$ holds if and only if

$$\Psi_0+\tau_k(\Psi_1+\Psi_2)<0\tag{19}$$

and

$$\Psi_0+\eta\Psi_1+\tau_k\Psi_2<0.\tag{20}$$

Furthermore, for $\tau_k\in[\tau_{\mathrm{m}},\ \tau_{\mathrm{M}}]$, inequality (19) holds if and only if

$$\Psi_0+\tau_{\mathrm{m}}(\Psi_1+\Psi_2)<0\tag{21}$$

and

$$\Psi_0+\tau_{\mathrm{M}}(\Psi_1+\Psi_2)<0.\tag{22}$$

Similarly, inequality (20) holds if and only if

$$\Psi_0+\eta\Psi_1+\tau_{\mathrm{m}}\Psi_2<0\tag{23}$$

and

$$\Psi_0+\eta\Psi_1+\tau_{\mathrm{M}}\Psi_2<0.\tag{24}$$

Thus, from the Schur complement, it is known that $\dot{V}(z(t))<0$ holds for $t\in[t_k,\ t_{k+1})(k\in N)$ if LMIs (13)–(16) hold. Since $V(z(t_k^+))\leqslant V(z(t_k^-))$, so the globally asymptotical stability of system (5) can be guaranteed if LMIs (13)–(16) hold.

The proof is completed.

Remark 2 In Theorem 1, a sufficient condition of globally asymptotical stability for the origin of system (5) is given in terms of solutions to a set of LMIs. Here, three techniques are employed. First, a new type of LKF is defined, which captures the sampling characteristic of network systems. Then, the coupled relationship (18) of terms τ_k and $t-t_k$ is clarified. It should be noted that $0\leqslant t-t_k\leqslant\eta-\tau_{\mathrm{m}}$ was used in [23–24] , which shows that the upper bound of $t-t_k$ in the k-th executive period is obviously enlarged. Finally, an iterative two-step convex combination technique is utilized, and an LMIs-based criterion is obtained.

Remark 3 Different from [15, 18–19] and [16], the sampling characteristic of network systems and the coupled relationship of terms τ_k and $t-t_k$ is taken into consideration, the new stability criterion in Theorem 1 is less conservative, which will be verified by an example in the sequel.

For sampled-data-based NN (10), one can obtain a stability criterion from Theorem 1 directly. For reducing the computational complexity, we will provide a less complex stability criterion. To this end, a reciprocally convex combination technique [20] will be used, so it is listed as the following lemma.

Lemma 1 For given positive semi-definite matrices Z_1 and Z_2, if there exists a matrix T such that

$$\begin{bmatrix} Z_1 & T \\ * & Z_2 \end{bmatrix} \geqslant 0,$$

then the following inequality for any $\alpha (0 < \alpha < 1)$ holds

$$\begin{bmatrix} \frac{1}{\alpha} Z_1 & 0 \\ * & \frac{1}{1-\alpha} Z_2 \end{bmatrix} \geqslant \begin{bmatrix} Z_1 & T \\ * & Z_2 \end{bmatrix}. \tag{25}$$

Proof It is ready to see that

$$\begin{bmatrix} \frac{1}{\alpha} Z_1 & 0 \\ * & \frac{1}{1-\alpha} Z_2 \end{bmatrix} - \begin{bmatrix} Z_1 & T \\ * & Z_2 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1-\alpha}{\alpha} Z_1 & -T \\ * & \frac{\alpha}{1-\alpha} Z_2 \end{bmatrix} =$$

$$\begin{bmatrix} \beta I & 0 \\ * & -\beta^{-1} I \end{bmatrix} \begin{bmatrix} Z_1 & T \\ * & Z_2 \end{bmatrix} \begin{bmatrix} \beta I & 0 \\ * & -\beta^{-1} I \end{bmatrix} \geqslant 0,$$

where $\beta = \sqrt{\frac{1-\alpha}{\alpha}}$.

Thus, the result is established.

For (10), we can choose an LKF candidate as

$$\begin{aligned} \tilde{V}(z(t)) = {} & V_1(z(t)) + \tilde{V}_2(z(t)) + \\ & V_3(z(t)) + V_4(z(t)), \end{aligned} \tag{26}$$

where $V_1(z(t)),\ V_3(z(t))$ and $V_4(z(t))$ are defined in (12) and

$$\tilde{V}_2(z(t)) = (\eta - (t - t_k)) \int_{t_k}^{t} \dot{z}^{\mathrm{T}}(s) R \dot{z}(s) \mathrm{d}s.$$

Theorem 2 For a given scalar $\eta > 0$, the origin of system (10) under Assumption 2 is globally asymptotically stable if there exist matrices $P = P^{\mathrm{T}} > 0$, $U = U^{\mathrm{T}} \geqslant 0$, $R = R^{\mathrm{T}} > 0$, $Z = Z^{\mathrm{T}} > 0$, $\Lambda_j = \mathrm{diag}\{\lambda_{j1}, \lambda_{j2}, \cdots, \lambda_{jn}\} \geqslant 0$, $D_j = \mathrm{diag}\{d_{j1}, d_{j2}, \cdots, d_{jn}\} \geqslant 0 (j = 1, 2)$, and matrices N_1 and N_2 with appropriate dimensions, such that

$$\begin{bmatrix} \Omega_1 & \mathcal{B}^{\mathrm{T}} R - \frac{1}{\eta} (\tilde{e}_1 - \tilde{e}_2)^{\mathrm{T}} N_2 & \eta \mathcal{B}^{\mathrm{T}} Z \\ * & -\frac{1}{\eta} R & 0 \\ * & * & -\eta Z \end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} R + Z & N_1 & N_2 \\ * & Z & 0 \\ * & * & R \end{bmatrix} \geqslant 0, \tag{28}$$

where

$$\begin{aligned} \Omega_1 = {} & \Omega_0 - \frac{1}{\eta} (\tilde{e}_1 - \tilde{e}_2)^{\mathrm{T}} (R + Z)(\tilde{e}_1 - \tilde{e}_2) - \\ & \frac{1}{\eta} (\tilde{e}_2 - \tilde{e}_3)^{\mathrm{T}} Z (\tilde{e}_2 - \tilde{e}_3) - \\ & \frac{1}{\eta} \mathrm{He}((\tilde{e}_1 - \tilde{e}_2)^{\mathrm{T}} N_1 (\tilde{e}_2 - \tilde{e}_3)), \\ \Omega_0 = {} & \mathrm{He}(\tilde{e}_1^{\mathrm{T}} (P + \Sigma \Lambda_2) \mathcal{B} + \tilde{e}_4^{\mathrm{T}} (\Lambda_1 - \Lambda_2) \mathcal{B} - \\ & \tilde{e}_4^{\mathrm{T}} D_1 \tilde{e}_4 + \tilde{e}_4^{\mathrm{T}} D_1 \Sigma \tilde{e}_1 - \tilde{e}_5^{\mathrm{T}} D_2 \tilde{e}_5 + \\ & \tilde{e}_5^{\mathrm{T}} D_2 \Sigma \tilde{e}_2) + \tilde{e}_1^{\mathrm{T}} U \tilde{e}_1 - \tilde{e}_3^{\mathrm{T}} U \tilde{e}_3, \end{aligned}$$

$$\begin{aligned} & \mathcal{B} = [-C \ \ 0 \ \ 0 \ \ A \ \ B],\ \tilde{e}_1 = [I \ \ 0 \ \ 0 \ \ 0 \ \ 0], \\ & \tilde{e}_2 = [0 \ \ I \ \ 0 \ \ 0 \ \ 0],\ \tilde{e}_3 = [0 \ \ 0 \ \ I \ \ 0 \ \ 0], \\ & \tilde{e}_4 = [0 \ \ 0 \ \ 0 \ \ I \ \ 0],\ \tilde{e}_5 = [0 \ \ 0 \ \ 0 \ \ 0 \ \ I]. \end{aligned}$$

Proof Since

$$\begin{aligned} \dot{\tilde{V}}_2(z(t)) = {} & (\eta - (t - t_k)) \xi^{\mathrm{T}}(t) \mathcal{B}^{\mathrm{T}} R \mathcal{B} \xi(t) - \\ & \int_{t_k}^{t} \dot{z}^{\mathrm{T}}(s) R \dot{z}(s) \mathrm{d}s, \end{aligned}$$

and from the Jensen's integral inequality [25], it gets that

$$\begin{aligned} & -\int_{t_k}^{t} \dot{z}^{\mathrm{T}}(s) R \dot{z}(s) \mathrm{d}s - \int_{t-\eta}^{t} \dot{z}^{\mathrm{T}}(s) Z \dot{z}(s) \mathrm{d}s \leqslant \\ & -\frac{1}{t - t_k} \xi^{\mathrm{T}}(t) (\tilde{e}_1 - \tilde{e}_2)^{\mathrm{T}} (R + Z)(\tilde{e}_1 - \tilde{e}_2) \xi(t) - \\ & \frac{1}{\eta - (t - t_k)} \xi^{\mathrm{T}}(t) (\tilde{e}_2 - \tilde{e}_3)^{\mathrm{T}} Z (\tilde{e}_2 - \tilde{e}_3) \xi(t), \end{aligned}$$

where

$$\xi(t) = [z^{\mathrm{T}}(t) \ \ z^{\mathrm{T}}(t_k) \ \ z^{\mathrm{T}}(t - \eta) \ \ \zeta_2^{\mathrm{T}}(t)]^{\mathrm{T}}.$$

Similar to the proof of Theorem 1, it yields that

$$\dot{\tilde{V}}(z(t)) \leqslant \xi^{\mathrm{T}}(t) \Omega(t) \xi(t), \tag{29}$$

where

$$\begin{aligned} \Omega(t) = {} & \Omega_0 + \eta \mathcal{B}^{\mathrm{T}} Z \mathcal{B} + (\eta - (t - t_k)) \mathcal{B}^{\mathrm{T}} R \mathcal{B} - \\ & \frac{1}{t - t_k} (\tilde{e}_1 - \tilde{e}_2)^{\mathrm{T}} (R + Z)(\tilde{e}_1 - \tilde{e}_2) - \\ & \frac{1}{\eta - (t - t_k)} (\tilde{e}_2 - \tilde{e}_3)^{\mathrm{T}} Z (\tilde{e}_2 - \tilde{e}_3). \end{aligned} \tag{30}$$

From the Schur complement, it is ready to see that $\Omega(t) < 0$ is equivalent to

$$\begin{aligned} & \begin{bmatrix} \Omega_0 + \eta \mathcal{B}^{\mathrm{T}} Z \mathcal{B} & \mathcal{B}^{\mathrm{T}} R \\ * & 0 \end{bmatrix} - \frac{1}{\eta} \begin{bmatrix} \tilde{e}_1 - \tilde{e}_2 & 0 \\ \tilde{e}_2 - \tilde{e}_3 & 0 \\ 0 & I \end{bmatrix}^{\mathrm{T}} \times \\ & \Omega_2(t) \begin{bmatrix} \tilde{e}_1 - \tilde{e}_2 & 0 \\ \tilde{e}_2 - \tilde{e}_3 & 0 \\ 0 & I \end{bmatrix} < 0, \end{aligned} \tag{31}$$

where

$$\Omega_2(t) = \begin{bmatrix} \frac{\eta}{t - t_k} (R + Z) & 0 \\ * & \frac{\eta}{\eta - (t - t_k)} \begin{bmatrix} Z & 0 \\ * & R \end{bmatrix} \end{bmatrix}.$$

So, from Lemma 1, one can find that $\Omega(t) < 0$ holds if

$$\begin{bmatrix} \Omega_0 + \eta \mathcal{B}^{\mathrm{T}} Z \mathcal{B} & \mathcal{B}^{\mathrm{T}} R \\ * & 0 \end{bmatrix} - \frac{1}{\eta} \begin{bmatrix} \tilde{e}_1 - \tilde{e}_2 & 0 \\ \tilde{e}_2 - \tilde{e}_3 & 0 \\ 0 & I \end{bmatrix}^{\mathrm{T}} \times \begin{bmatrix} R + Z & N_1 & N_2 \\ * & Z & 0 \\ * & * & R \end{bmatrix} \begin{bmatrix} \tilde{e}_1 - \tilde{e}_2 & 0 \\ \tilde{e}_2 - \tilde{e}_3 & 0 \\ 0 & I \end{bmatrix} < 0, \quad (32)$$

which implies that the globally asymptotical stability of system (10) can be guaranteed if LMIs (27)−(28) hold.

Thus, the proof is completed.

Remark 4 By using a reciprocally convex combination technique [20], a less complex stability criterion for sampled-data-based NNs is proposed in Theorem 2. Here, fewer variables are involved.

4 Numerical example

Consider the network-based NN (1) with a time-varying delay and [15]

$$C = \mathrm{diag}\{1.2769,\ 0.6231,\ 0.9230,\ 0.4480\},$$

$$A = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$\sigma_1 = 0.1137,\ \sigma_2 = 0.1279,$$
$$\sigma_3 = 0.7994,\ \sigma_4 = 0.2368.$$

Case 1 $\tau_{\mathrm{M}} = 0$. In this case, network-based NN (5) reduces to a sampled-data-based NN (10), and η determines an upper bound on the sampling intervals. The upper bounds given by [6, 15–16, 19] and our Theorems 1 and 2 are listed in Table 1, which shows that our results are better than the existing ones.

Table 1 Calculated upper bounds of η.

Methods	η
[6]	1.2598
[15]	2.0389
[19]	2.0389
[16]	2.0770
Theorem 1	2.3451
Theorem 2	2.3451

Case 2 $\tau_{\mathrm{M}} > 0$. For this case, Table 2 presents a comparison of the corresponding upper bounds of η for different τ_{m} and τ_{M} derived by the methods in [19] and Theorem 1.

Table 2 Upper bounds of η for different τ_{m} and τ_{M}

τ_{M}	0.2		0.5		1	
τ_{m}	0	0.2	0	0.5	0	1
[19]	2.038	2.038	2.038	2.038	2.038	2.043
Theorem 1	2.184	2.185	2.114	2.115	2.078	2.081

From Tables 1 and 2, it is clear that larger allowable upper bounds of η can be obtained by Theorems 1 and 2, which shows the benefits of the proposed method.

5 Conclusions

In this paper, the delay-dependent stability problem of network-based NNs has been investigated. By defining an appropriate Lyapunov functional and clarifying the coupled relationship of network-induced delay and the executive duration, a new less conservative delay-dependent stability criterion has been derived in terms of LMIs. Meanwhile, by employing a reciprocally convex combination approach, a delay-dependent stability criterion with less complexity has also been proposed for sampled-data-based NNs. A numerical example has been given to illustrate the effectiveness of the presented criteria and the improvement over the existing results.

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