第 30 卷第 6 期 控 制 理 论 与 应 用 Vol. 30 No. 6
2013 年 6 月 Control Theory & Applications Jun. 2013

DOI: 10.7641/CTA.2013.12137

布尔控制网络的能控性与能观性

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摘要: 利用矩阵的半张量积, 布尔控制网络被转化为离散时间系统. 本文从离散时间系统的结构矩阵出发, 讨论了逻辑控制系统的能控能观性条件, 得到了一个新的能控性条件. 新的条件简化了原有能控性矩阵的计算复杂性, 矩阵的最高阶数由原来的2^{m+n} 降到了2^n. 另外, 还得到了检验布尔控制网络能观性的条件. 与原有条件相比, 新的条件更容易计算检验. 最后, 给出一个实例, 检验给出的能控能观性判断条件的正确性.

关键词: 布尔控制网络; 能控性; 能观性; 矩阵半张量积
中图分类号: TP273 **文献标识码**: A

Controllability and observability of Boolean control networks

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Abstract: Using the semi-tensor product, we convert the Boolean control network to its algebraic form. From the structure matrix of Boolean control network, the controllability and observability of the Boolean control network are discussed. A novel necessary and sufficient condition for controllability, which improves the recent results, is given. The new controllability condition eliminates the redundant computation of controllability matrix. The highest power of matrix is reduced from 2^{m+n} to 2^n. Also, a sufficient condition for observability is obtained, which can be computed easily. A numerical example is presented to show the applicability of our controllability and observability condition. condition.

Key words: Boolean control network; controllability; observability; semi-tensor product

1 Introduction

In order to investigate the gene expression, Kauffman firstly introduced Boolean networks[1]. The study of Boolean networks has attracted a great attention from biologists, physicists, and social scientists, because it provides a simple and proper model to describe artificial intelligent systems, neuronal networks, and genomic regulatory networks[2–4]. The most important problem about Boolean networks is to find its topological structure, including fixed points, cycles, basin of attractors, and the transient time[5–7]. The control problems of Boolean networks has received much attention[8–9]. Almost all the results of Boolean networks are about random (probabilistic) Boolean network, because the study of probabilistic Boolean networks can be converted to a markovian chain. While classical markovian chain theories provide many analytical results to study the various problems about probabilistic Boolean networks[9].

Recently, using semi-tensor product, algebraic state space representation of Boolean control networks (BCNs) is introduced by Cheng and Qi[5]. This representation is proved to be quite useful for studying BCNs. By investigating the corresponding discrete time systems, some interesting results have been obtained, including the controllability and observability of Boolean networks[10–11], the realization of Boolean networks[12], and the stability and stabilization design of Boolean networks[13], the decoupling of Boolean control networks[14], the optimal control problems[15–16]. Meanwhile, the Boolean networks with time delay is studied in [17]. In [18], the controllability of probabilistic Boolean control networks is discussed. The monograph about semi-tensor product method to Boolean network is published by Springer[19]. For more details we refer the reader to a tutorial survey[19–20] and references therein.

In [10], a systematic method of controllability and observability of Boolean control networks has been developed. The problems of both controllability and observability are solved by giving necessary and sufficient conditions. In [20–21], the conditions for controllability and observability are improved. But the computation is still redundant. The purpose of this paper is:

1) To eliminate the redundant computation of controllability matrix. The highest power of matrix used in [10, 20–21] is 2^{m+n}, while in Theorems 5 and 6, the

Received 17 June 2012; revised 25 September 2012.
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This work is supported by the National Natural Science Foundation of China (Nos. 61203050, 61143002, 61104065), and the Natural Science Foundation of Educational Commission of Henan Province (Nos. 2011B110002, 12B110002).

highest power of the same matrix is reduced to 2^n;

2) To give a sufficient condition for observability of BCNs. In Theorem 8, a sufficient condition is given to test the observability, which can be easily obtained.

The remainder of the paper is organized as follows. In Section 2, some necessary preliminaries are given. In Section 3, an improved controllability matrix is constructed, and also an easily computable matrix is given to test the observability. Section 4 gives a numerical example to illustrate the effectiveness of our conditions. Finally, a brief summary is given in Section 5.

2 Preliminaries

In this section, we first list the notations used in this paper, and review the related results about how to convert the Boolean control network to its algebraic form.

Notations:

· Let δ_n^i be the i-th column of the identity matrix I_n, and $\varDelta_n := \{\delta_n^1, \delta_n^2, \cdots, \delta_n^n\}$. When $n = 2$ we simply use $\varDelta := \varDelta_2$.

· The set of logical values: True ($T \sim 1$) and False ($F \sim 0$), is denoted by $\mathcal{D} = \{0, 1\}$. We identify each with a vector as $T \sim \delta_2^1$ and $F \sim \delta_2^2$. So in vector form the set of logical values becomes $\varDelta$. In this sense we have the equivalence as $\mathcal{D} \sim \varDelta$.

· Assume a matrix $M = [\delta_n^{i_1}\ \delta_n^{i_2}\ \cdots\ \delta_n^{i_s}] \in M_{n\times s}$, i.e., its columns, $\mathrm{Col}(M) \subset \varDelta_n$. We call M a logical matrix, and simply denote it as

$$M = \delta_n[i_1\ i_2\ \cdots\ i_s].$$

· The set of $n \times s$ logical matrices is denoted by $\mathcal{L}_{n\times s}$.

· A matrix $B \in M_{n\times s}$ is called a Boolean matrix, if its entries $b_{ij} \in \mathcal{D}, \forall\, i, j$.

· The set of $n \times s$ Boolean matrices is denoted by $\mathcal{B}_{n\times s}$.

· A matrix $B = (b_{ij}) \gg 0$ means $b_{ij} > 0$.

· A vector $\alpha = (a_i) \gg 0$ means $a_i > 0$.

In vector form, the logical function can be expressed as an algebraic function.

Theorem 1[22] Let $x_1, \cdots, x_n \in \mathcal{D}$ be n logical variables, and $f(x_1, \cdots, x_n)$ a logical function. Then there exists a unique matrix $M_f \in \mathcal{L}_{2\times 2^n}$, called the structure matrix of f, such that in vector form we have

$$f(x_1, \cdots, x_n) = M_f \ltimes_{i=1}^n x_i,\ x_i \in \varDelta.$$

In [10], the Boolean network with additional inputs and outputs is discussed. Its dynamics can be expressed as follows:

$$\begin{cases} x_1(t+1) = \\ f_1(x_1(t), \cdots, x_n(t), u_1(t), \cdots, u_m(t)), \\ \quad\vdots \\ x_n(t+1) = \\ f_n(x_1(t), \cdots, x_n(t), u_1(t), \cdots, u_m(t)), \\ y_j(t) = h_j(x_1(t), \cdots, x_n(t)),\ j = 1, \cdots, q. \end{cases} \tag{1}$$

where $x_i(t) \in \varDelta$ are logical variables, f_i, $i = 1, \cdots, n$, and h_j, $j = 1, \cdots, q$ are logical functions, $u_i(t) \in \varDelta$, $i = 1, \cdots, m$ are controls.

Set $x = \ltimes_{i=1}^n x_i$, $u = \ltimes_{i=1}^m u_i$, $y = \ltimes_{i=1}^q y_i$. Then $x \in \varDelta_{2^n}$, $u \in \varDelta_{2^m}$, $y \in \varDelta_{2^q}$. Using vector form, the algebraic form of (1) is denoted as

$$x(t+1) = Lu(t)x(t), \tag{2}$$

$$y(t) = Hx(t), \tag{3}$$

where $L \in \mathcal{L}_{2^n\times 2^{n+m}}$, and $H \in \mathcal{B}_{2^q\times 2^n}$.

3 Main results

In this section, the conditions for observability and controllability are given by the structure matrix of Eq.(2). The structure matrix of BCN (2) is equally split as

$$\begin{aligned} L = [\mathrm{Blk}_1(L)\ \ \mathrm{Blk}_1(L)\ \ \cdots\ \ \mathrm{Blk}_{2^m}(L)] = \\ [B_1\ B_2\ \cdots\ B_{2^m}], \end{aligned} \tag{4}$$

where $\mathrm{Blk}_i(L)$ is the i-th block of matrix L, and $B_i = \mathrm{Blk}_i(L) \in \mathcal{L}_{2^n\times 2^n}$, $i = 1, \cdots, 2^m$.

Define

$$M = L \ltimes \mathbf{1}_{2^m} \in \mathcal{L}_{2^n\times 2^n}. \tag{5}$$

3.1 Controllability of BCNs

Definition 1[20] BCN (2) is controllable from x_0 to x_{d}, if there exist a $T > 0$, and a sequence of controls $\{u(0), u(1), \cdots, u(T-1)\}$, such that the trajectory of Eq.(2) starting from x_0 can be driven by the controls to $x(T) = x_{\mathrm{d}}$. System (2) is controllable at x_0, if it is controllable to any $x_{\mathrm{d}} \in \varDelta_{2^n}$. System (2) is controllable, if it is controllable at any x_0.

Define

$$C = \sum_{k=1}^{2^{m+n}} M^k. \tag{6}$$

The matrix C is called the controllability matrix in [20–21]. Using input-state incidence matrix, the following result about controllability of system (2) is given in [11, 20].

Theorem 2[11,20] Consider BCN (2).

i) Starting from $x_0 = \delta_{2^n}^j$, $x_{\mathrm{d}} = \delta_{2^n}^i$ is reachable, if and only if, $c_{ij} > 0$;

ii) System (2) is controllable from $x_0 = \delta_{2^n}^j$, if and only if, $\mathrm{Col}_j(C) \gg 0$;

iii) System (2) is controllable, if and only if, $C \gg 0$.

In Theorem 2, to verify the controllability of Eq.(2), the matrix C should be obtained. In fact, the computation matrix C in Eq.(6) is redundant. Define

$$M_C = \sum_{k=1}^{2^n} M^k. \tag{7}$$

We will prove that the matrix M_C given in Eq.(7) is enough to test the controllability. In [11], the physical meaning of nonzero entries in matrix M^k is explained. When $k = 1$, from Eq.(5), we know that M_{ij} means whether there exist a control sequence steering state $\delta_{2^n}^j$ to $\delta_{2^n}^i$ in one step by judging if $M_{ij} = 1$ or not. In this paper, we will give an alternative proof to the following theorem, which will be useful for our result.

Theorem 3 For BCN (2), the matrix M is obtained from structure matrix L by Eq. (5). Suppose $(M^s)_{i,j} = c$. Then there are c different paths from state $\delta_{2^n}^j$ to $\delta_{2^n}^i$ at s-th step with proper control sequence.

Proof We prove it by mathematical induction. When $s = 1$ the conclusion follows from the construction of matrix M given in Eq.(5). Now assume $(M^s)_{ij}$ is the number of the paths from $\delta_{2^n}^j$ to $\delta_{2^n}^i$ at s-th step. Since a path from $\delta_{2^n}^j$ to $\delta_{2^n}^i$ at $(s+1)$-th step can always be considered as a path from $\delta_{2^n}^j$ to $\delta_{2^n}^k$ at s-th step and then from $\delta_{2^n}^k$ to $\delta_{2^n}^i$ at one step. The different ways from $\delta_{2^n}^j$ to $\delta_{2^n}^i$ at $(s+1)$-th step can be calculated by

$$N_{s+1}(j,i) = \sum_{k=1}^{2^n} N_s(j,k) \times N_1(k,i), \tag{8}$$

where $N_s(j,k)$ denotes the number of different ways from $\delta_{2^n}^j$ to $\delta_{2^n}^k$ at s-th step, and $N_1(k,i)$ denotes the number of different ways from $\delta_{2^n}^k$ to $\delta_{2^n}^i$ at one step.

It is obvious that

$$N_s(j,k) = (M^s)_{kj}. \tag{9}$$

Next, we consider $N_1(k,i)$. Suppose $(\mathrm{Blk}_p(L))_{ik} = 1$, then we can choose control $\delta_{2^m}^p$, such that $\delta_{2^n}^i = L\delta_{2^m}^p\delta_{2^n}^k$. Hence, $N_1(k,i)$ is the number of p, which satisfies $(\mathrm{Blk}_p(L))_{ik} = 1$. It is said that

$$N_1(k,i) = (\sum_{\alpha=1}^{2^m} \mathrm{Blk}_\alpha(L))_{ik} = M_{ik}. \tag{10}$$

Substitute Eq.(8) with Eqs.(9) and (10), then

$$N_{s+1}(j,i) = \sum_{k=1}^{2^n} (M^s)_{kj} \times M_{ik}. \tag{11}$$

The right hand Eq.(11) can be considered as product of the i-th row of M and the j-th column of M^s. From Eqs.(9)−(11), $(M^{s+1})_{i,j}$ is the number of the paths from $\delta_{2^n}^j$ to $\delta_{2^n}^i$ at $(s+1)$-th step. The conclusion is proved.

Theorem 4 Consider BCN (2). For any two different states $\alpha, \beta \in \varDelta_{2^n}$, if $\beta = \delta_{2^n}^i$ can be reached from $\alpha = \delta_{2^n}^j$ within at most s-steps, then β can be reached from α within 2^n-steps by proper control sequence.

Proof If $1 \leqslant s \leqslant 2^n$, then we are done. Otherwise, suppose $\alpha = \delta_{2^n}^j$ is reached from $\beta = \delta_{2^n}^i$ at s-th $(s > 2^n)$ step with the following trajectory:

$$\begin{cases} \delta_{2^n}^{i_1} = L\delta_{2^m}^{\gamma_0}\beta, \\ \delta_{2^n}^{i_2} = L\delta_{2^m}^{\gamma_1}\delta_{2^n}^{i_1}, \\ \quad\vdots \\ \delta_{2^n}^{i_p} = L\delta_{2^m}^{\gamma_{p-1}}\delta_{2^n}^{i_{p-1}}, \\ \delta_{2^n}^{i_{p+1}} = L\delta_{2^m}^{\gamma_p}\delta_{2^n}^{i_p}, \\ \quad\vdots \\ \delta_{2^n}^{i_q} = L\delta_{2^m}^{\gamma_{q-1}}\delta_{2^n}^{i_{q-1}}, \\ \delta_{2^n}^{i_{q+1}} = L\delta_{2^m}^{\gamma_q}\delta_{2^n}^{i_q}, \\ \quad\vdots \\ \delta_{2^n}^{i_s} = L\delta_{2^m}^{\gamma_{s-1}}\delta_{2^n}^{i_{s-1}} = \alpha. \end{cases} \tag{12}$$

In sequence $\delta_{2^n}^{i_1}, \cdots, \delta_{2^n}^{i_s}$, if there exist two states $\delta_{2^n}^{i_p} = \delta_{2^n}^{i_q}$. Choosing control $u(p) = \delta_{2^m}^{\gamma_q}$ at p-th step, yields

$$\delta_{2^n}^{i_{q+1}} = L\delta_{2^m}^{\gamma_q}\delta_{2^n}^{i_p}.$$

Then the trajectory from $\beta = \delta_{2^n}^j$ to $\alpha = \delta_{2^n}^i$ becomes

$$\beta \to \delta_{2^n}^{i_1} \to \cdots \to \delta_{2^n}^{i_p} \to \delta_{2^n}^{i_{q+1}} \to \cdots \to \delta_{2^n}^i = \alpha.$$

Using the above process, we can exclude all the repeated states in sequence $\delta_{2^n}^{i_1}, \cdots, \delta_{2^n}^{i_s}$. Without loss of generality, suppose there doesnot exist any repeated state in sequence

$$\delta_{2^n}^j = \beta \to \delta_{2^n}^{i_1} \to \cdots \to \delta_{2^n}^{i_s} = \alpha = \delta_{2^n}^i. \tag{13}$$

In BCN (2), there are 2^n different states. Hence, the length of sequence (13) is no larger than 2^n. It is said that from $\beta = \delta_{2^n}^j$ to $\alpha = \delta_{2^n}^i$, there exists at least one trajectory sequence with length no larger than 2^n. The conclusion is proved.

In matrix M_C, we can obtain all the reachability information of any two states within 2^n steps. To test the controllability of Eq.(2), the matrix M_C is enough. It may be the real simplest controllability matrix. From Theorem 4, Theorem 2 can be improved as

Theorem 5 Consider BCN (2).

i) Starting from $x_0 = \delta_{2^n}^j$, $x_\mathrm{d} = \delta_{2^n}^i$ is reachable, if and only if, $(M_C)_{ij} > 0$;

ii) System (2) is controllable from $x_0 = \delta_{2^n}^j$, if and only if, $\mathrm{Col}_j(M_C) \gg 0$;

iii) System (2) is controllable, if and only if, $M_C \gg 0$.

In fact, we do not need to consider the true number of matrix M_C. What we really need is: whether it is positive or zero. Borrowing from characteristic function, construct matrix $\bar{M} = (\bar{m}_{ij})$ from matrix $M = (m_{ij})$, where

$$\bar{M}_{ij} = \begin{cases} 0, \ m_{ij} = 0, \\ 1, \ m_{ij} > 0. \end{cases}$$

For Boolean matrix, the Boolean product is defined as follows.

Definition 2[11,23] 1) If $a, b \in D$, we can define the Boolean addition and the Boolean product respectively as

$$a +_\mathcal{B} b = a \vee b, \ a \times_\mathcal{B} b = a \wedge b.$$

2) Let $A \in \mathcal{B}_{m\times n}$ and $B \in \mathcal{B}_{n\times p}$. Then $A \times_\mathcal{B} B := C \in \mathcal{B}_{m\times p}$ as

$$c_{ij} = a_{i1} \times_\mathcal{B} b_{1j} +_\mathcal{B} \cdots +_\mathcal{B} a_{in} \times_\mathcal{B} b_{nj}.$$

Partially, let $A \in \mathcal{B}_{n\times n}$. Then

$$A^{(2)} := A \times_\mathcal{B} A.$$

Using Boolean product, construct characteristic matrices of $\bar{M}_C$ as

$$\bar{M}_C = \sum_{i=1\ \mathcal{B}}^{2^n} M^{(i)}. \tag{14}$$

Now we restate Theorem 5 as following.

Theorem 6 BCN (2) is globally controllable, if and only if, $\bar{M}_C \gg 0$.

3.2 Observability of BCNs

The observability of BCNs is considered in [10].

Definition 3 [10] BCN (2) is said to be observable if for any initial state x_0 there exists at least a Boolean sequence of control, such that the initial state can be determined by the output sequence.

The observability matrix is constructed and the necessary and sufficient condition is given in [10, 19].

In [20–21], the necessary and sufficient condition is given by an alternative one. Denote

$$\Gamma_1=\begin{pmatrix} HB_1 \\ HB_2 \\ \vdots \\ HB_{2^m} \end{pmatrix},\ \Gamma_2=\begin{pmatrix} HB_1B_1 \\ HB_1B_2 \\ \vdots \\ HB_{2^m}B_{2^m} \end{pmatrix},\cdots. \quad (15)$$

Using them, the observability matrix is constructed as

$$\mathcal{O}_1=[\Gamma_0^{\mathrm{T}}\ \ \Gamma_1^{\mathrm{T}}\ \ \Gamma_2^{\mathrm{T}}\ \ \cdots\ \ \Gamma_{s^*}^{\mathrm{T}}]^{\mathrm{T}}, \quad (16)$$

where $\Gamma_0=H$, and s^* is the smallest positive integer such that $\Gamma_{s^*+1}\subset\bigcup_{k=1}^{s^*}\Gamma_k$. For more details we refer the reader to [21]

Theorem 7 [21] Assume BCN (2) is globally controllable, then Eq.(2) is observable, if and only if $\operatorname{rank}\mathcal{O}_1=2^n$.

To verify the observability of Eq.(2) with output (3), we have to obtain the observability matrix $\mathcal{O}_1$ in Eq.(16). The computation of $\mathcal{O}_1$ is complicated. In the following, we give an easily computable matrix used to test the observability. The times of matrix product is at most $1+2+\cdots+2^n-1$. Define

$$\mathcal{O}_2=[H^{\mathrm{T}}\ \ (HM)^{\mathrm{T}}\ \ (HM^2)^{\mathrm{T}}\ \ \cdots\ \ (HM^{2^n-1})^{\mathrm{T}}]^{\mathrm{T}}. \quad (17)$$

Theorem 8 Assume BCN (2) is globally controllable. If $\operatorname{rank}(\mathcal{O}_2)=2^n$, then (2) is observable.

Proof Based on matrix theory, it is easy to note

$$\operatorname{rank}\begin{pmatrix} H \\ HB_1 \\ HB_2 \\ \vdots \\ HB_{2^m} \\ \vdots \end{pmatrix} = \operatorname{rank}\begin{pmatrix} H \\ HB_1+\cdots+HB_{2^m} \\ HB_2 \\ \vdots \\ HB_{2^m} \\ \vdots \end{pmatrix}. \quad (18)$$

In fact $M=\sum_{i=1}^{2^m}B_i$, the power of M can be expressed as

$$\begin{aligned} M^2 = {} & (B_1+\cdots+B_{2^m})(B_1+\cdots+B_{2^m}) = \\ & B_1B_1+B_1B_2+\cdots+B_{2^m}B_{2^m} = \\ & \sum_{i=1}^{2^m}\sum_{j=1}^{2^m}B_iB_j, \\ & \vdots \\ M^s = {} & (B_1+\cdots+B_{2^m})^s = \\ & \sum_{i_1=1}^{2^m}\cdots\sum_{i_s=1}^{2^m}B_{i_s}\cdots B_{i_s}, \\ & \vdots \end{aligned}$$

From right hand side of Eq.(18), yields

$$\operatorname{rank}(\mathcal{O}_1)\geqslant\operatorname{rank}\begin{pmatrix} H \\ HM \\ HM^2 \\ \vdots \\ HM^{2^n-1} \end{pmatrix}=\operatorname{rank}(\mathcal{O}_2). \quad (19)$$

There are 2^n columns in matrices $\mathcal{O}_1$ and $\mathcal{O}_2$. Hence,

$$\operatorname{rank}(\mathcal{O}_2)\leqslant\operatorname{rank}(\mathcal{O}_1)\leqslant 2^n.$$

When $\operatorname{rank}(\mathcal{O}_2)=2^n$, yields $\operatorname{rank}(\mathcal{O}_1)=2^n$. If BCN (2) is controllable, from Theorem 7, we can conclude that (2) is observable.

4 An illustrative example

Reconsider the Example 29 given in [10]. Consider the following system:

$$\begin{cases} A(t+1)=B(t)\leftrightarrow C(t), \\ B(t+1)=C(t)\vee u_1(t), \\ C(t+1)=A(t)\wedge u_2(t), \end{cases} \quad (20)$$

with the outputs

$$\begin{aligned} y_1(t)&=A(t), \\ y_2(t)&=B(t)\vee C(t). \end{aligned} \quad (21)$$

Set

$$x(t)=A\ltimes B(t)\ltimes C(t),$$
$$y(t)=y_1(t)\ltimes y_2(t)$$

and $u(t)=u_1(t)\ltimes u_2(t)$. The algebraic form of Eqs. (20) and (21) is

$$\begin{aligned} x(t+1)&=Lu(t)x(t), \\ y(t)&=Hx(t), \end{aligned} \quad (22)$$

where $L\in\mathcal{L}_{8\times 32}$, which is

$$\begin{aligned} L=\delta_8[&1\ 5\ 5\ 1\ 2\ 6\ 6\ 2\ 2\ 6\ 6\ 2\ 2\ 6\ 6\ 2 \\ &1\ 7\ 5\ 3\ 2\ 8\ 6\ 4\ 2\ 8\ 6\ 4\ 2\ 8\ 6\ 4], \end{aligned}$$

and $H\in\mathcal{M}_{4\times 8}$ is

$$H=\delta_4[1\ 1\ 1\ 2\ 3\ 3\ 3\ 4].$$

We can obtain

$$M = L \ltimes \mathbf{1}_4 = \begin{pmatrix} 2&0&0&1&0&0&0&0\\ 2&0&0&1&4&0&0&2\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&2\\ 0&1&2&0&0&0&0&0\\ 0&1&2&0&0&2&4&0\\ 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&2&0&0 \end{pmatrix}.$$

The characteristic matrix about M is obtained as

$$\bar{M} = \begin{pmatrix} 1&0&0&1&0&0&0&0\\ 1&0&0&1&1&0&0&1\\ 0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&1\\ 0&1&1&0&0&0&0&0\\ 0&1&1&0&0&1&1&0\\ 0&1&0&0&0&0&0&0\\ 0&1&0&0&0&1&0&0 \end{pmatrix}.$$

The controllability matrix is computed as

$$\bar{M}_C = \sum_{i=1\ \mathcal{B}}^{2^n} \bar{M}^{(i)} = (a_{ij}),$$

where $\forall i, j, a_{ij} = 1$.

From Theorem 6, the BCN (22) is controllable. To verify the observability of (22), we compute $HM^k, k = 1, \cdots, 2^n - 1$. The observability matrix $\bar{\mathcal{O}}$ is obtained as

$$\mathcal{O} = \begin{pmatrix} H \\ HM^1 \\ HM^2 \\ \vdots \\ HM^{2^n-1} \end{pmatrix} = \begin{pmatrix} 1&1&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&1&1&0\\ 0&0&0&0&0&0&0&1\\ 4&0&0&3&4&0&0&2\\ 0&0&0&1&0&0&0&2\\ 0&3&4&0&0&2&4&0\\ 0&1&0&0&0&2&0&0\\ 8&6&8&7&0&4&0&6\\ 0&2&0&1&0&4&0&2\\ 6&6&4&7&12&4&8&6\\ 2&2&4&1&4&4&8&2\\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots \end{pmatrix}.$$

From the first 12 rows of $\mathcal{O}$, we can obtain $\mathrm{rank}(\mathcal{O}) = 8$. From Theorem 8, we can conclude that BCN (20) with outputs (21) is observable.

5 Conclusion

In this paper, we eliminated the redundant computation of controllability matrix and obtained an easily computable matrix to test observability of BCNs. For controllability of BCNs (2), the necessary and sufficient condition given in [10] is improved. Also a new observability matrix is obtained for testing observability of BCNs (2). We should point out that the condition for observability in this paper is not necessary, while it is easily computable.

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