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离散时间非线性系统的数据驱动无模型自适应迭代学习控制

金尚泰[1], 侯忠生[1], 池荣虎[2], 柳向斌[1]

(1. 北京交通大学 电子信息工程学院 先进控制系统研究所, 北京 100044; 2. 青岛科技大学 自动化与电气工程学院, 山东 青岛 266042)

摘要: 本文基于迭代域的动态线性化方法, 提出了一类单入单出离散时间非线性系统的数据驱动无模型自适应迭代学习控制方案. 无模型自适应迭代学习控制本质上属于一种数据驱动控制方法, 仅利用被控对象的输入输出数据即可实现控制方案的设计. 理论分析表明无模型自适应迭代学习控制方案可以保证最大学习误差的单调收敛性. 数值仿真和快速路交通控制应用验证了无模型自适应迭代学习控制方案的有效性.

关键词: 数据驱动控制; 迭代学习控制; 无模型自适应控制; 动态线性化方法; 单调收敛性; 快速路交通控制

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Data-driven model-free adaptive iterative learning control for a class of discrete-time nonlinear systems

JIN Shang-tai[1], HOU Zhong-sheng[1], CHI Rong-hu[2], LIU Xiang-bin[1]

(1. Advanced Control Systems Laboratory, School of Electronic and Information Engineering,
Beijing Jiaotong University, Beijing 100044, China;
2. School of Automation and Electrical Engineering, Qingdao University of Science and Technology, Qingdao Shandong 266042, China)

Abstract: In this paper, a data-driven model-free adaptive iterative learning control (MFAILC) scheme is proposed based on a novel dynamic linearization approach along the iteration axis for a class of repetitive discrete-time single input single output (SISO) nonlinear systems. The MFAILC is essentially a data-driven control method that designs controller merely using the measured input and output data of the controlled plant. Theoretical analysis shows that the MFAILC guarantees the monotonic convergence of the iteration maximum error. Numerical example and freeway traffic control application are given to illustrate the effectiveness of the MFAILC.

Key words: data-driven control; iterative learning control; model-free adaptive control; dynamic linearization approach; monotonic convergence; freeway traffic control

1 Introduction

Iterative learning control (ILC) was originally proposed as an intelligent learning mechanism for robot manipulators[1]. The basic idea of ILC is to update the control signal of the present operation cycle by feeding back the tracking error from the previous cycle for improving the control performance. Because of its simplicity and efficiency, ILC has attracted much attention in the past two decades[2–6].

The convergence analysis of ILC is an important theoretical issue. Up to now, three typical analysis methods, namely contraction mapping based ILC, composite energy function (CEF) based ILC, and optimization based ILC, have been formulated. Most ILC methods utilize the contraction mapping and fixed point theory to design linear iterative learning algorithm[7–8]. And the pointwise convergence property of tracking error over finite time interval is achieved in the sense of λ norm with the requirements on global Lipschitz and identical initialization conditions. CEF based ILC relaxes the global Lipschitz condition by introducing the information the system states[9–11]. For local Lipschitz nonlinear systems, the asymptotic convergence of tracking error along the iteration axis is guaranteed by Lyapunov analysis method under the identical initialization conditions. The first two methods do not require the information of the state space model. Thus, they can be regarded as the data-driven or model free control approach. However, the limitation of these two methods lies in the fact that the transient performance of the system output along the iteration axis becomes poor sometimes. Consequently, the application of these two methods is limited. Optimization based ILC is proposed for the known linear systems[12–16]. The explicit optimal cost function is given and minimized to design optimal ILC algorithm, and monotonic tracking error convergence along the iteration axis is guaranteed. Compared with the first two methods, optimization based

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ILC overcomes the limitation of poor transient performance along the iteration axis. However, it requires the accurate known linear model of the controlled plant and identical condition for designing ILC algorithm. As we know, the model of practical plants is very difficult to establish, and sometimes it is impossible in the view of cost. Thus, the study of data-driven model-free optimal iterative learning control for the unknown discrete-time nonlinear systems has important significance. Then, we can enjoy not only extra good properties of the norm optimal ILC, but also the little requirements on the system dynamic model of the prototype of the ILC as well. Here, the 'data-driven model free control' means that the controller design is merely using measured input and output (I/O) data of a plant without explicitly using any information from mathematical model of the controlled plant, and whose stability, convergence and robustness can be guaranteed by rigorous mathematic analysis under certain reasonable assumptions.

Now, a few data-driven control approaches could be found, such as: iterative feed-back tuning, virtual reference feedback tuning, and iterative learning control, model-free adaptive control, etc[17–18]. Model-free adaptive control (MFAC) was proposed for a class of discrete-time nonlinear systems[19–22] based on a series of the dynamic linearization methods. The main feature of the approach is that the controller design depends merely on measured I/O data of the controlled plant. The theoretical analysis and comparison experiments show that the approach guarantees the bounded input and bounded output (BIBO) stability and tracking error monotonic convergence[21–22]. MFAC approach has been successfully implemented in many practical applications[23–28].

In this paper, a data-driven model-free adaptive ILC (MFAILC) is proposed to deal with repetitive discrete-time SISO nonlinear systems. The main contributions of this paper are that: 1) The controller design depends merely on the measured I/O data of the controlled plant, while the existing norm optimal ILC methods are based on the known model. 2) MFAILC relax the identical initial condition, while the majority of ILC schemes require the identical initial condition. 3) The monotonic convergence of MFAILC in maximum-norm sense is proved. 4) The effectiveness of the proposed MFAILC is verified by numerical example and freeway traffic control application.

This paper is organized as follows. In Section 2, partial form dynamic linearization (PFDL) approach is proposed for a class of repetitive discrete-time SISO nonlinear systems. In Section 3, MFAILC scheme is designed based on PFDL approach, and its properties are discussed. In Section 4, the numerical example and freeway traffic control application are given to verify the effectiveness of the proposed scheme. Finally, some conclusions are drawn in Section 5.

2 Dynamic linearization approach for repetitive discrete-time SISO nonlinear systems

Consider the following repetitive discrete-time SISO nonlinear system:

$$\begin{aligned} y_k(t+1) = f(\,& y_k(t), \cdots, y_k(t-n_{\mathrm{y}}), \\ & u_k(t), \cdots, u_k(t-n_{\mathrm{u}})), \end{aligned} \tag{1}$$

where $y_k(t), u_k(t)$ are the output and input at time instant t of k-th iteration, respectively. $t = 0, 1, \cdots, N-1$, $k \in \mathbb{Z}_+$, N is a finite positive integer. $n_{\mathrm{y}}, n_{\mathrm{u}}$ are two unknown integers. $f(\cdot)$ is an unknown nonlinear function.

Denote $\boldsymbol{U}_k(t)$ as a vector consisting of all control input signals within a moving time window $[t-L+1, t]$,

$$\boldsymbol{U}_k(t) = (u_k(t), \cdots, u_k(t-L+1))^{\mathrm{T}}$$

with $\boldsymbol{U}_k(t) = \mathbf{0}$ for $t < 0$. The integer L is called control input linearization length constant, and $\mathbf{0}$ is the zero vector of dimension L.

Two assumptions are made on system (1) before partial form dynamic linearization (PFDL) approach is elaborated.

Assumption 1 The partial derivatives of $f(\cdot)$ with respect to control inputs $u_k(t), \cdots, u_k(t-L+1)$ are continuous.

Assumption 2 Suppose that $\forall t \in \{0, 1, \cdots, N-1\}$ and $\forall k \in \mathbb{Z}_+$, when $\|\Delta \boldsymbol{U}_k(t)\| \neq 0$, system (1) satisfies generalized Lipschitz condition, that is

$$|\Delta y_k(t+1)| \leqslant b \|\Delta \boldsymbol{U}_k(t)\|, \tag{2}$$

where $\Delta y_k(t+1) = y_k(t+1) - y_{k-1}(t+1)$, $\Delta \boldsymbol{U}_k(t) = \boldsymbol{U}_k(t) - \boldsymbol{U}_k(t-1)$, and b is a finite positive constant.

Remark 1 From a practical point of view, these assumptions imposed on the plant are reasonable and acceptable. Assumption 1 is a typical condition for many control methods which a general nonlinear system should satisfy. Assumption 2 imposes an upper bound limitation on change rate of the system output driven by changes of the control inputs. From an energy viewpoint, the energy change inside a system cannot go to infinity if the change of the control input energy is at a finite level. Many practical systems, such as servo control system, temperature control system, pressure control system, traffic control system, etc., satisfy this assumption.

Lemma 1 Consider nonlinear system (1) satisfying Assumption 1−2. For any fixed L, if $\|\Delta \boldsymbol{U}_k(t)\| \neq \mathbf{0}$, then there exists an iteration-dependent time-varying parameter $\boldsymbol{\theta}_k(t)$, called pseudo gradient (PG), such that system (1) can be transformed into the following PFDL data model in iteration domain,

$$\Delta y_k(t+1) = \boldsymbol{\theta}_k^{\mathrm{T}}(t) \Delta \boldsymbol{U}_k(t), \tag{3}$$

with bounded $\boldsymbol{\theta}_k(t) = [\theta_{1,k}(t) \ \cdots \ \theta_{L,k}(t)]^{\mathrm{T}}$.

Proof From Eq.(1) we have

$$\begin{aligned}
&\Delta y_k(t+1) = \\
&f(y_k(t),\cdots,y_k(t-n_{\mathrm{y}}),u_k(t),\cdots,u_k(t-n_{\mathrm{u}}))- \\
&f(y_k(t),\cdots,y_k(t-n_{\mathrm{y}}),u_{k-1}(t),\cdots, \\
&u_{k-1}(t-L+1),u_k(t-L),\cdots,u_k(t-n_{\mathrm{u}}))+ \\
&f(y_k(t),\cdots,y_k(t-n_{\mathrm{y}}),u_{k-1}(t),\cdots, \\
&u_{k-1}(t-L+1),u_k(t-L),\cdots,u_k(t-n_{\mathrm{u}}))- \\
&f(y_{k-1}(t),\cdots,y_{k-1}(t-n_{\mathrm{y}}), \\
&u_{k-1}(t),\cdots,u_{k-1}(t-n_{\mathrm{u}}))= \\
&\frac{\partial f^*}{\partial u_k(t)}\Delta u_k(t)+\cdots+ \\
&\frac{\partial f^*}{\partial u_k(t-L+1)}\Delta u_k(t-L+1)+\psi_k(t), \qquad (4)
\end{aligned}$$

where $\dfrac{\partial f^*}{\partial u_k(t-i)}$ represents the partial derivative value of function f with respect to $u_k(t-i)$ at some point in the interval $[u_k(t-i), u_{k-1}(t-i)]$, $i = 0,1,\cdots,L-1$, and

$$\begin{aligned}
\psi_k(t) = f(&y_k(t),\cdots,y_k(t-n_{\mathrm{y}}),u_{k-1}(t), \\
&\cdots,u_{k-1}(t-L+1),u_k(t-L), \\
&\cdots,u_k(t-n_{\mathrm{u}}))-f(y_{k-1}(t),\cdots, \\
&y_{k-1}(t-n_{\mathrm{y}}),u_{k-1}(t),\cdots,u_{k-1}(t-n_{\mathrm{u}})).
\end{aligned}$$

Consider following equation with a variable $\boldsymbol{\eta}_k(t)$:

$$\psi_k(t) = \boldsymbol{\eta}_k^{\mathrm{T}}(t)\Delta \boldsymbol{U}_k(t). \qquad (5)$$

Since $\|\Delta \boldsymbol{U}_k(t)\| \neq \mathbf{0}$, there at least exists a solution $\boldsymbol{\eta}_k^*(t)$ to Eq.(5).

Let

$$\begin{aligned}
&\boldsymbol{\theta}_k(t) = [\theta_{1,k}(t)\ \cdots\ \theta_{L,k}(t)]^{\mathrm{T}} = \\
&[\frac{\partial f^*}{\partial u_k(t)}\cdots\frac{\partial f^*}{\partial u_k(t-L+1)}]^{\mathrm{T}} + \boldsymbol{\eta}_k^*(t). \qquad (6)
\end{aligned}$$

Eq.(4) can be rewritten as

$$\Delta y_k(t+1) = \boldsymbol{\theta}_k^{\mathrm{T}}(t)\Delta \boldsymbol{U}_k(t).$$

This gives the main conclusion of the theorem. And the secondary concern of the boundedness of PG is guaranteed directly by using Assumption 2 and Eq.(3).

Remark 2 Lemma 1 is an extension of the results of references[19–21]. From the proof of the Lemma 1, we can see that $\boldsymbol{\theta}_k(t)$ is related with input and output signals till time instant t of $(k-1)$-th and k-th iterations. Thus, it is an iteration-dependent time-varying parameter. On the other hand, $\boldsymbol{\theta}_k(t)$ can be considered as a differential signal in some sense and it is bounded for any t and k. If $\|\Delta \boldsymbol{U}_k(t)\|$ is not too large, $\boldsymbol{\theta}_k(t)$ may be regarded as a slowly iteration-varying parameter. Consequently, we can implement adaptive iterative learning control of the original system by designing a parameter estimator to estimate the parameter.

Remark 3 Consider the linear time-invariant (LTI) system

$$\begin{cases} x(t+1) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \qquad (7)$$

where $t = 0,\cdots,N-1$.

The system output in k-th iteration is

$$y_k(t+1) = CA^{t+1}x_k(0) + \sum_{i=0}^{t-1} CA^{t-i}Bu_k(i), \qquad (8)$$

and the system output in $(k-1)$-th iteration is

$$y_{k-1}(t+1) = CA^{t+1}x_{k-1}(0) + \sum_{i=0}^{t-1} CA^{t-i}Bu_{k-1}(i). \qquad (9)$$

Obviously, if $x_k(0) = x_{k-1}(0)$, system (7) can be rewritten as

$$\Delta y_k(t+1) = \sum_{i=0}^{t-1} CA^{t-i}B\Delta u_k(i), \qquad (10)$$

where $\Delta y_k(t+1) = y_k(t+1) - y_{k-1}(t+1)$ and $\Delta u_k(i) = u_k(i) - u_{k-1}(i)$.

In this case, the PG $\boldsymbol{\theta}_k(t)$ is just the Markov sequence of the system if $L = N$, that is $\boldsymbol{\theta}_k(t) = [CB\ CAB\ \cdots\ CA^{N-1}B]^{\mathrm{T}}$. It is same as [12–15].

3 MFAILC system design and stability analysis

For a given desired trajectory $y_{\mathrm{d}}(t),\ t = 0,1,\cdots,N-1$, the control objective of ILC is to design a sequence of appropriate control inputs $u_k(t)$ such that the tracking error $e_k(t+1) = y_{\mathrm{d}}(t+1) - y_k(t+1)$ converges to zero as the iteration number k approaches infinity.

Define the optimization design problem for the ILC controller as follows:

$$\min_{u_k(t)} J_k(u_k(t)) = \min_{u_k(t)}(|e_k(t+1)|^2 + \lambda\|\Delta \boldsymbol{U}_k(t)\|^2), \qquad (11)$$

subjected to

$$e_k(t+1) = e_{k-1}(t+1) - \boldsymbol{\theta}_k^{\mathrm{T}}(t)\Delta \boldsymbol{U}_k(t), \qquad (12)$$

where $\lambda > 0$ is a weighting factor.

Remark 4 The use of optimization objective in iterative learning control is not new to this paper. In [12], an optimization objective introduced explicitly for repetitive discrete-time system.

Using the optimal condition $\dfrac{\partial J}{\partial u_k(t)} = 0$, we have

$$\begin{aligned}
u_k(t) = {} & u_{k-1}(t) + \frac{\rho_1\theta_{1,k}(t)e_{k-1}(t+1)}{\lambda + |\theta_{1,k}(t)|^2} - \\
& \frac{\sum_{i=2}^{L}\rho_i\theta_{1,k}(t)\theta_{i,k}(t)\Delta u_k(t-i+1)}{\lambda + |\theta_{1,k}(t)|^2}, \qquad (13)
\end{aligned}$$

where $0 < \rho_i \leqslant 1$, $i \in \{1,\cdots,L\}$ is the step-size factor added.

Since $\boldsymbol{\theta}_k(t)$ is unknown, the following cost function of parameter estimation is used to derive the estimator:

$$\begin{aligned}
J(\hat{\boldsymbol{\theta}}_k^{\mathrm{T}}(t)) = {} & |\Delta y_{k-1}(t+1) - \hat{\boldsymbol{\theta}}_k^{\mathrm{T}}(t)\Delta \boldsymbol{U}_{k-1}(t)|^2 + \\
& \mu\|\hat{\boldsymbol{\theta}}_k^{\mathrm{T}}(t) - \hat{\boldsymbol{\theta}}_{k-1}^{\mathrm{T}}(t)\|^2, \qquad (14)
\end{aligned}$$

where $\mu > 0$ is a weighting factor, $\hat{\boldsymbol{\theta}}_k(t)$ is the estimation value of $\boldsymbol{\theta}_k(t)$.

The minimization of above criterion function gives estimation algorithm

$$\hat{\boldsymbol{\theta}}_k(t) = \hat{\boldsymbol{\theta}}_{k-1}(t) + \frac{\eta\Delta\boldsymbol{U}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{U}_{k-1}(t)\|^2} \times (\Delta y_{k-1}(t+1) - \hat{\boldsymbol{\theta}}^{\mathrm{T}}_{k-1}(t)\Delta\boldsymbol{U}_{k-1}(t)), \quad (15)$$

where $\eta \in (0,2)$ is a step factor added.

Correspondingly, the learning control law (13) is rewritten as

$$u_k(t) = u_{k-1}(t) + \frac{\rho_1\hat{\theta}_{1,k}(t)e_{k-1}(t+1)}{\lambda + |\hat{\theta}_{1,k}(t)|^2} - \frac{\sum\limits_{i=2}^{L}\rho_i\hat{\theta}_{1,k}(t)\hat{\theta}_{i,k}(t)\Delta u_k(t-i+1)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}. \quad (16)$$

In order to endow the parameter estimation algorithm (15) with a strong ability to track time-varying parameter, we present a reset algorithm as follows:

$$\hat{\boldsymbol{\theta}}_k(t) = \hat{\boldsymbol{\theta}}_0(t), \text{ if } \operatorname{sgn}(\hat{\theta}_{1,k}(t)) \neq \operatorname{sgn}(\hat{\theta}_{1,0}(t)) \text{ or } \|\hat{\boldsymbol{\theta}}_k(t)\| \leqslant \epsilon \text{ or } \|\Delta\boldsymbol{U}_{k-1}(t)\| \leqslant \epsilon, \quad (17)$$

where ϵ is a small positive constant, $\hat{\boldsymbol{\theta}}_0(t)$ is the initial value of $\hat{\boldsymbol{\theta}}_k(t)$.

Remark 5 PFDL data model based MFAILC (PFDL-MFAC) scheme for nonlinear system (1) is constructed by integrating learning control law (16), parameter updating law (15), and reset algorithm (17). The proposed scheme has L-dimensional vector to be updated on-line, and is quite different from the traditional adaptive control system, in which there are usually $2n$ model parameters to be estimated on-line, where n denotes the order of the controlled plant.

Remark 6 PFDL-MFAILC scheme only utilizes I/O data of the controlled plant to design the controller. So it is regarded as a data-driven control approach. It is worth pointing out that the PG estimation $\hat{\boldsymbol{\theta}}_k(t)$ affects the learning gains in control law (16) virtually and can be iteratively calculated by estimation law (15) and reset algorithm (17), which is quite different from standard ILC, where its learning gain is fixed and cannot be tuned automatically and iteratively.

Theorem 1 Suppose Assumptions 1−2 hold, system (1) is controlled by learning control law (16), parameter updating law (15) and reset algorithm (17), then $\hat{\boldsymbol{\theta}}_k(t)$ is bounded for all $t \in [0, N-1]$ and $k \in \mathbb{Z}_+$.

Proof If $\operatorname{sgn}(\hat{\theta}_{1,k}(t)) \neq \operatorname{sgn}(\hat{\theta}_{1,0}(t))$, or $\|\hat{\boldsymbol{\theta}}_k(t)\| \leqslant \epsilon$ or $\|\Delta\boldsymbol{U}_{k-1}(t)\| \leqslant \epsilon$, then the boundedness of $\hat{\boldsymbol{\theta}}_k(t)$ is obvious.

In other case, define PG estimation error as

$$\tilde{\boldsymbol{\theta}}_k(t) = \hat{\boldsymbol{\theta}}_k(t) - \boldsymbol{\theta}_k(t). \quad (18)$$

Subtracting $\boldsymbol{\theta}_k(t)$ from both sides of Eq.(15) and using Eq.(3), we have

$$\tilde{\boldsymbol{\theta}}_k(t) = \frac{(I - \eta\Delta\boldsymbol{U}_{k-1}(t)\Delta\boldsymbol{U}^{\mathrm{T}}_{k-1}(t))\tilde{\boldsymbol{\theta}}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{U}_{k-1}(t)\|^2} + \boldsymbol{\theta}_{k-1}(t) - \boldsymbol{\theta}_k(t). \quad (19)$$

From Lemma 1, we can see that $\boldsymbol{\theta}_k(t)$ is bounded by a positive constant $\bar{b}$, which leads to $\|\boldsymbol{\theta}_{k-1}(t) - \boldsymbol{\theta}_k(t)\| \leqslant 2\bar{b}$.

Taking norms on both sides of Eq.(19), yields

$$\|\tilde{\boldsymbol{\theta}}_k(t)\| \leqslant \|\frac{(I - \eta\Delta\boldsymbol{U}_{k-1}(t)\Delta\boldsymbol{U}^{\mathrm{T}}_{k-1}(t))\tilde{\boldsymbol{\theta}}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{U}_{k-1}(t)\|^2}\| + 2\bar{b}. \quad (20)$$

Let $\Xi_k(t) = \dfrac{(I - \Delta\boldsymbol{U}_{k-1}(t)\Delta\boldsymbol{U}^{\mathrm{T}}_{k-1}(t))\tilde{\boldsymbol{\theta}}_{k-1}(t)}{\mu + \|\Delta\boldsymbol{U}_{k-1}(t)\|^2}$, computing $\|\Xi_k(t)\|^2$ yields

$$\|\Xi_k(t)\|^2 = \|\tilde{\boldsymbol{\theta}}_{k-1}(t)\|^2 + \frac{\|\eta\Delta\boldsymbol{U}^{\mathrm{T}}_{k-1}(t)\tilde{\boldsymbol{\theta}}_{k-1}(t)\|^2}{\mu + ||\Delta\boldsymbol{U}_{k-1}(t)||^2} \times (-2 + \frac{\eta||\Delta\boldsymbol{U}_{k-1}(t)||^2}{\mu + ||\Delta\boldsymbol{U}_{k-1}(t)||^2}). \quad (21)$$

Since $\boldsymbol{\theta}_k(t)$ is an iteration dependent column vector and $0 < \eta < 2$, $\mu > 0$, we have

$$\|\Xi_k(t)\|^2 < \|\tilde{\boldsymbol{\theta}}_{k-1}(t)\|^2. \quad (22)$$

Apparently, there exists $0 < d_1 < 1$ such that

$$\|\tilde{\boldsymbol{\theta}}_k(t)\| \leqslant d_1^k||\tilde{\boldsymbol{\theta}}_0(t)|| + 2\bar{b}(1 - d_1^k)/(1 - d_1). \quad (23)$$

Note that, we only need the existence of constant d_1 instead of its exact value.

Eq.(23) implies $\tilde{\boldsymbol{\theta}}_k(t)$ bounded. From Theorem 1, we have $\boldsymbol{\theta}_k(t)$ is bounded, so $\hat{\boldsymbol{\theta}}_k(t)$ is bounded.

For the stability and convergence of the MFAILC scheme (15) and (17), another assumption should be made.

Assumption 3 The first element of PG $\theta_{1,k}(t) > \xi$ (or $\theta_{1,k}(t) < -\xi$) for all k, where ξ is a positive constant. Without loss of generality, we consider $\theta_{1,k}(t) > \xi$.

Remark 7 This assumption is similar to the limitation of control input direction in reference [29]. In fact, many practical systems can satisfy this assumption. For the freeway traffic control system in following section, the traffic flow density will increase (or not decrease at least) when the on-ramp metering traffic volume increases in practice.

Lemma 2[30] Let $A = \begin{bmatrix} a_1 & a_2 & \cdots & a_{L-1} & a_L \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$. If $\sum\limits_{i=1}^{L}|a_i| < 1$, then $s(A) < 1$, where $s(A)$ is the spectral radius of A.

Theorem 2 If nonlinear system (1), satisfying Assumptions 1−3, is controlled by PFDL-MFAILC scheme (15)−(17), then there exists $\lambda_{\min} > 0$ such that the following two properties hold for any $\lambda > \lambda_{\min}$:

a) The maximum learning error $e_{\max}(k) = \max\limits_{t\in\{1,\cdots,N\}} e_k(t)$ converges to zero as k approaches to infinity, that is $\lim\limits_{k\to\infty} |e_{\max}(k)| = 0$.

b) $u_k(t)$ and $y_k(t)$ are bounded for all $t \in \{1, \cdots, N\}$ and $k \in \mathbb{Z}_+$.

Proof From Lemma 1, we can see that $\boldsymbol{\theta}_k(t)$ is bounded by a positive constant $\bar{b}$, which leads to $\|\boldsymbol{\theta}_k(t)\| \leqslant \bar{b}$. There exists $\lambda_{\min} > 0$, such that following inequalities (24)−(27) hold when $\lambda > \lambda_{\min}$,

$$|\frac{\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}| \leqslant |\frac{\hat{\theta}_{1,k}(t)}{2\sqrt{\lambda}|\hat{\theta}_{1,k}(t)|}| < \frac{1}{2\sqrt{\lambda_{\min}}} = M_1 < \frac{0.5}{\bar{b}}, \tag{24}$$

$$0 < M_2 \leqslant |\frac{\theta_{1,k}(t)\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}| \leqslant \bar{b}|\frac{\hat{\theta}_{1,k}(t)}{2\sqrt{\lambda}|\hat{\theta}_{1,k}(t)|}| < \frac{\bar{b}}{2\sqrt{\lambda_{\min}}} < 0.5, \tag{25}$$

$$\|\boldsymbol{\theta}_k(t)\|_v \leqslant M_3, \tag{26}$$

$$(\sum_{i=2}^{L} |\frac{\hat{\theta}_{1,k}(t)\hat{\theta}_{i,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}|)^{\frac{1}{L-1}} \leqslant M_4. \tag{27}$$

Selecting $\max\limits_{i\in\{2,\cdots,L\}} \rho_i$ such that

$$\sum_{i=2}^{L} \rho_i |\frac{\hat{\theta}_{1,k}(t)\hat{\theta}_{i,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}| \leqslant (\max_{i\in\{2,\cdots,L\}} \rho_i) \sum_{i=2}^{L} |\frac{\hat{\theta}_{1,k}(t)\hat{\theta}_{i,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}| \leqslant (\max_{i\in\{2,\cdots,L\}} \rho_i) M_4^{L-1} < 1. \tag{28}$$

Let

$$A_k(t) = \begin{bmatrix} -\frac{\rho_2\hat{\theta}_{1,k}(t)\hat{\theta}_{2,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2} \cdots -\frac{\rho_L\hat{\theta}_{1,k}(t)\hat{\theta}_{L,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2} & 0 \\ I & \mathbf{0} \end{bmatrix}$$

and $C = [1\ 0\ \cdots\ 0]^{\mathrm{T}}$, then Eq.(16) can be rewritten as

$$\Delta\boldsymbol{U}_k(t) = A_k(t)\Delta\boldsymbol{U}_k(t-1) + \frac{\rho_1\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2} C e_{k-1}(t+1). \tag{29}$$

The characteristic equation of $A_k(t)$ is

$$z^L + \frac{\rho_2\hat{\theta}_{1,k}(t)\hat{\theta}_{2,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2} z^{L-1} + \cdots + \frac{\rho_L\hat{\theta}_{1,k}(t)\hat{\theta}_{L,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2} z = 0. \tag{30}$$

From Eq.(28) and Lemma 2 we have $|z| < 1$. Thus, the following inequality holds

$$|z|^{L-1} \leqslant \sum_{i=2}^{L} \rho_i |\frac{\hat{\theta}_{1,k}(t)\hat{\theta}_{i,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}||z|^{L-i} \leqslant \sum_{i=2}^{L} \rho_i |\frac{\hat{\theta}_{1,k}(t)\hat{\theta}_{i,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}| \leqslant (\max_{i\in\{2,\cdots,L\}} \rho_i) M_4^{L-1}. \tag{31}$$

Inequality (31) implies that

$$|z| \leqslant (\max_{i\in\{2,\cdots,L\}} \rho_i)^{\frac{1}{L-1}} M_4 < 1. \tag{32}$$

Note that, there exists an arbitrary small positive constant ϵ_1 such that the following inequality holds:

$$\|A_k(t)\|_v \leqslant s(A_k(t)) + \varepsilon_1 \leqslant (\max_{i\in\{2,\cdots,L\}} \rho_i)^{\frac{1}{L-1}} M_4 + \varepsilon_1 < 1, \tag{33}$$

where $s(A_k(t))$ is the spectral radius of $A_k(t)$, and $\|A_k(t)\|_v$ is the consistent matrix norm of $A_k(t)$.

Let $d_2 = (\max\limits_{i\in\{2,\cdots,L\}} \rho_i)^{\frac{1}{L-1}} M_4 + \varepsilon_1$. Taking norm on both sides of Eq.(29), we have

$$\begin{aligned} &\|\Delta\boldsymbol{U}_k(t)\|_v \leqslant \\ &\|A_k(t)\|_v \|\Delta\boldsymbol{U}_k(t-1)\|_v + \\ &\rho_1|\frac{\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}||e_{k-1}(t+1)| < \\ &d_2\|\Delta\boldsymbol{U}_k(t-1)\|_v + \rho_1 M_1|e_{k-1}(t+1)| < \\ &\vdots \\ &d_2^{t+1}\|\Delta\boldsymbol{U}_k(-1)\|_v + d_2^t\rho_1 M_1|e_{k-1}(1)| + \\ &\cdots + \rho_1 M_1|e_{k-1}(t+1)| = \\ &\rho_1 M_1 \sum_{i=0}^{t} d_2^{t-i}|e_{k-1}(i+1)|. \end{aligned} \tag{34}$$

From the definition of tracking error, PFDL data model (3), and Eq.(29), we have

$$\begin{aligned} &e_k(t+1) = y_{\mathrm{d}}(t+1) - y_k(t+1) = \\ &y_{\mathrm{d}}(t+1) - y_{k-1}(t+1) - \boldsymbol{\theta}_k^{\mathrm{T}}(t)\Delta\boldsymbol{U}_k(t) = \\ &(1 - \frac{\rho_1\theta_{1,k}(t)\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2})e_{k-1}(t+1) - \\ &\boldsymbol{\theta}_k^{\mathrm{T}}(t)A_k(t)\Delta\boldsymbol{U}_k(t-1). \end{aligned} \tag{35}$$

Taking norm on both sides of Eq.(35), we have

$$|e_k(t+1)| < |1 - \frac{\rho_1\theta_{1,k}(t)\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}||e_{k-1}(t+1)| + d_2\|\boldsymbol{\theta}_k^{\mathrm{T}}(t)\|_v\|\Delta\boldsymbol{U}_k(t-1)\|_v. \tag{36}$$

Selecting $0 < \rho_1 \leqslant 1$ such that

$$|1 - \rho_1\frac{\theta_{1,k}(t)\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}| = |1 - \rho_1|\frac{\theta_{1,k}(t)\hat{\theta}_{1,k}(t)}{\lambda + |\hat{\theta}_{1,k}(t)|^2}|| \leqslant 1 - \rho_1 M_2 < 1.$$

Let $d_3 = 1 - \rho_1 M_2$. Using Eq.(26), Eq.(36) can be rewritten as

$$|e_k(t+1)| < d_3|e_{k-1}(t+1)| + d_2 M_3\|\Delta\boldsymbol{U}_k(t-1)\|_v. \tag{37}$$

Substituting Eq.(34) into Eq.(37) yields

$$|e_k(t+1)| < d_3|e_{k-1}(t+1)| + \rho_1 M_1 M_3 \sum_{i=1}^{t-1} d_2^{t-1-i}|e_{k-1}(i+1)|. \tag{38}$$

Assume $e_{\max}(k) = e_k(\tau_1)$, then the following inequality holds:

$$\begin{aligned}&|e_{\max}(k)| = |e_k(\tau_1)| < \\ &d_3|e_{k-1}(\tau_1)|+\rho_1 M_1 M_3 \sum_{i=1}^{\tau_1-2} d_2^{\tau_1-2-i}|e_{k-1}(i+1)| \leqslant \\ &d_3|e_{\max}(k-1)| + \\ &\rho_1 M_1 M_3 \sum_{i=1}^{\tau_1-2} d_2^{\tau_1-2-i} e_{\max}(k-1) < \\ &(d_3 + \rho_1 M_1 M_3 \frac{d_2}{1-d_2})|e_{\max}(k-1)|.\end{aligned} \tag{39}$$

Selecting $0 < \rho_1 \leqslant 1, \cdots, 0 < \rho_L \leqslant 1$ such that

$$\begin{aligned}&d_3 + \rho_1 M_1 M_3 \frac{d_2}{1-d_2} = \\ &1 - \rho_1 M_2 + \rho_1 M_1 M_3 \frac{d_2}{1-d_2} = \\ &1 - \rho_1(M_2 + M_1 M_3) + \frac{\rho_1 M_1 M_3}{1-d_2} = \\ &1 - \rho_1(M_2 + M_1 M_3) + \\ &\frac{\rho_1 M_1 M_3}{1 - (\max\limits_{i\in\{2,\cdots,L\}} \rho_i)^{\frac{1}{L-1}} M_4 - \varepsilon_1} < 1.\end{aligned}$$

Let $d_4 = d_3 + \rho_1 M_1 M_3 \frac{d_2}{1-d_2}$. It is obviously that

$$\begin{aligned}&\lim_{k\to\infty} |e_{\max}(k)| < \lim_{k\to\infty} d_4|e_{\max}(k-1)| < \\ &\cdots < \lim_{k\to\infty} d_4^{k-1}|e_{\max}(1)| = 0.\end{aligned} \tag{40}$$

As $y_{\rm d}(t)$ is bounded, the convergence of $e_{\max}(k)$ implies that $y_k(t)$ is also bounded.

From Eqs. (34) and (40), we have

$$\begin{aligned}&\|\boldsymbol{U}_k(t)\|_v \leqslant \\ &\sum_{h=2}^{k} \|\boldsymbol{\Delta U}_h(t)\|_v + \|\boldsymbol{U}_1(t)\|_v < \\ &\rho_1 M_1 \sum_{h=2}^{k}\sum_{i=0}^{t} d_2^{t-i}|e_{h-1}(i+1)| + \|\boldsymbol{U}_1(t)\|_v < \\ &\frac{\rho_1 M_1}{1-d_2}\sum_{h=2}^{k} e_{\max}(h-1) + \|\boldsymbol{U}_1(t)\|_v < \\ &\frac{\rho_1 M_1 e_{\max}(1)}{(1-d_2)(1-d_4)} + \|\boldsymbol{U}_1(t)\|_v.\end{aligned} \tag{41}$$

This implies that $u_k(t)$ is bounded.

Compared with the existing results of norm optimal ILC, the proposed MFAILC scheme provides a more general and comprehensive framework for quadratic criterion based ILC, and has three distinct features: 1) coping with discrete-time SISO nonlinear system; 2) the system dynamic model is not required; 3) the strict identical initial condition is not required.

4 Simulations

In this section, two simulations are provided to verify the effectiveness of the proposed scheme.

4.1 Numerical simulation

Consider the nonlinear system

$$y(t+1)=\begin{cases}\dfrac{5y(t)y(t-1)}{1+y(t)^2+y(t-1)^2+y(t-2)^2}+ \\ u(t)+1.1u(t-1), \qquad t \leqslant 50, \\ \dfrac{2.5y(t)y(t-1)}{1+y(t)^2+y(t-1)^2}+ \\ 1.2u(t)+1.4u(t-1)+ \\ 0.7\sin(0.5(y(t)+y(t-1)))\cdot \\ \cos(0.5(y(t)+y(t-1))), \ t > 50.\end{cases}$$

The first subsystem in this example is taken from [31], and the second one is taken from [32]. Here the two subsystems are used to represent the unknown nonlinear system in a different time interval, where the structure, order and parameter of the controlled system are time-varying. Moreover, the first subsystem has not good performance using the neural network control method for its nonlinearity and non-minimum phase characteristics.

The desired trajectory is given as follows

$$y_{\rm d}(t+1) = 5\sin\frac{t\pi}{50} + 2\cos\frac{t\pi}{100}.$$

The initial output value $y_k(0)$ is randomly varying in the interval $[-0.05, 0.05]$ when the iteration i evolves. The control input of the first iteration is equal to 0. The parameters of the proposed scheme are $L = 2,\ \rho_1 = \rho_2 = 0.9,\ \lambda = 1,\ \mu = 1$.

In order to show that the proposed algorithm can overcome the limitations on the identical initial condition of traditional ILC, random varying initial value $y_k(0)$ is used. Fig.1 shows the profile of the initial value $y_k(0)$ over 100 iterations. Fig.2 shows the maximum learning error, where the horizon is the iteration number and the vertical axis is $\max\limits_{t\in\{1,\cdots,N\}} |e_k(t)|$. The effectiveness of the proposed ILC can be seen from Figs.1−2. Despite the random initial values (Fig.1) along the iteration axis, the tracking error converges asymptotically to zero along the iteration axis (Fig.2).

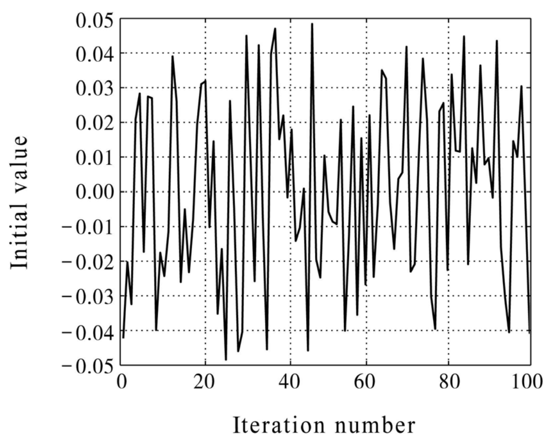

Fig. 1 The profile of random initial value

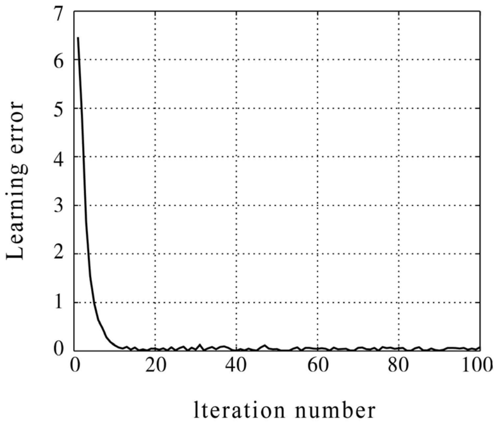

Fig. 2 $\max\limits_{t\in\{0,100\}} e_k(t)$ with the proposed MFAILC

4.2 Freeway traffic flow process

The macroscopic traffic flow patterns usually show repeated behavior on a daily or a weekly basis[33–34]. In this work, the freeway traffic flow process is simulated to verify the effectiveness of MFAILC. The macroscopic traffic flow model[33–34] for a single freeway with one on-ramp and one off-ramp on each section is as follows:

$$\rho_i(t+1) = \rho_i(t) + \frac{T}{L_i}(q_{i-1}(t) - q_i(t) + r_i(t) - s_i(t)), \tag{42}$$

$$q_i(t) = \rho_i(t)v_i(t), \tag{43}$$

$$v_i(t+1) = v_i(t) + \frac{T}{\tau}(V(\rho_i(t)) - v_i(t)) + \frac{T}{L_i}v_i(t)(v_{i-1}(t) - v_i(t)) - \frac{\nu T(\rho_{i+1}(t) - \rho_i(t))}{\tau L_i(\rho_i(t) + \kappa)}, \tag{44}$$

$$V(\rho_i(t)) = v_{\text{free}}(1 - (\frac{\rho_i(t)}{\rho_{\text{jam}}})^l)^m, \tag{45}$$

where T is the sample time interval, $t \in \{0, \cdots, N-1\}$ is t-th time interval, and $i \in \{1, \cdots, I\}$ is i-th section of the freeway, and I is the total section number. Model parameter variables are listed below:

$\rho_i(t)$: density in section i at time tT, (veh·(lane·km)$^{-1}$);

$v_i(t)$: space mean speed in section i at time tT, (km·h^{-1});

$q_i(t)$: traffic flow leaving section i and entering section $i+1$ at time tT, (veh·h^{-1});

$r_i(t)$: on-ramp traffic volume for section i at time tT, (veh·h^{-1});

$s_i(t)$: off-ramp traffic volume for section i at time tT, (veh·h^{-1}), which is regarded as an unknown disturbance;

L_i: length of freeway in section i, km;

V_{free}/(km·h^{-1}) and ρ_{jam}/(veh·(lane·km)$^{-1}$) are the free speed and the maximum possible density per lane, respectively.

τ/h, ν/(km^2·h^{-1}), κ/(veh·km^{-1}), l and m are constant parameters which reflect particular characteristics of a given traffic system and depend on the freeway geometry, vehicle characteristics, drivers' behaviors, etc.

Eq.(42) is the well-known conservation equation, Eq.(43) is the flow equation, Eq.(44) is the empirical dynamic speed equation, and Eq.(45) represents the density-dependent equilibrium speed.

We assume that the traffic flow rate entering section 1 during the time period tT and $(t+1)T$ is $q_0(t)$ and the mean speed of the traffic entering section 1 is equal to the mean speed of section 1, i.e. $v_0(t) = v_1(t)$. We also assume that the mean speed and traffic density of the traffic exiting section $I+1$ are equal to those of section I, i.e. $v_{I+1}(t) = v_I(t)$, $\rho_{I+1}(t) = \rho_I(t)$. Boundary conditions can be summarized as follows:

$$\rho_0(t) = q_0(t)/v_1(t), \tag{46}$$

$$v_0(t) = v_1(t), \tag{47}$$

$$\rho_{I+1}(t) = \rho_I(t), \tag{48}$$

$$v_{I+1}(t) = v_I(t). \tag{49}$$

Due to the highly nonlinear and uncertain nature of traffic flow model, a control profile cannot be calculated directly from the model. The MFAILC will be employed just depending on the input $r_i(t)$ and output $\rho_i(t)$ data of the freeway traffic system.

Consider a long segment of freeway that is divided uniformly into 12 sections. The length of each section is 0.5 km. The initial traffic volume entering section 1 is 1500 (veh·h^{-1}). The desired density is $\rho_{\text{d}}(t) = 30$ (veh·(lane·km)$^{-1}$). The initial density and mean speed of each section are shown in Table 1 and the parameters used in this model are also listed in Table 1[33–34].

Table 1 Initial values and parameters associated with the traffic model

Parameter	Value	Parameter	Value
v_{free}	80	ν	35
ρ_{jam}	80	$q_0(t)$	1500
κ	13	$r_i(0)$	1.8
τ	0.01	l	0
T	0.00417	m	1.7
α	0.95		

There exist an on-ramp with known traffic demands in Section 7 and an off-ramp with unknown exiting traffic flow in Section 4. The traffic demand pattern (on-ramp) and the outflow pattern (off-ramp) are shown in Fig.3. They were chosen to simulate a traffic scenario during rush hour. Note that the queuing demands actually impose a constraint on the control inputs of ramp metering, i.e., the on-ramp volumes cannot exceed the current demands plus the existing waiting queues at on-ramp 7 at time t, thus

$$r_7(t) \leqslant d_7(t) + l_7(t), \tag{50}$$

where $l_7(t)$ denotes the length (in vehicles) of a possibly existing waiting queue at time t at the 7th on-ramp, $d_7(t)$ is the demand flow at time t at 7th on-ramp(veh·h^{-1}).

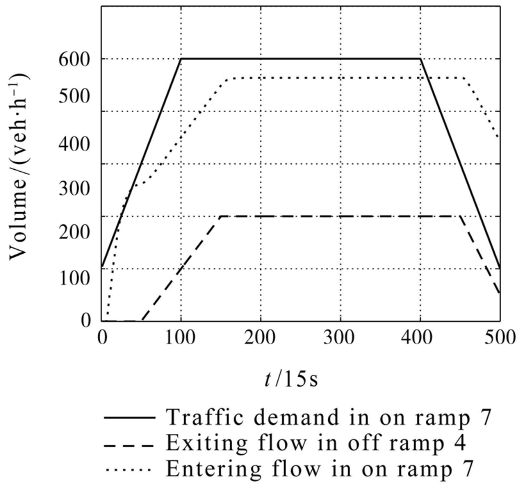

Fig. 3 Traffic demand in on-ramp 7, entering flow in on-ramp 7, and unknown exiting flow in off-ramp 4

On the other hand, the waiting queue is the accumulation of the difference between the demand and actual on-ramp, i.e.,

$$l_7(t+1) = l_7(t) + T(d_7(t) - r_7(t)). \tag{51}$$

In order to show the proposed algorithm can overcome the limitations on identical initial condition of traditional ILC, a random varying along iteration axis $\rho_7(0)$ is used (Fig.4). In the simulation we choose $L = 2, \rho_1 = \rho_2 = 0.9, \lambda = 0.001, \mu = 1$.

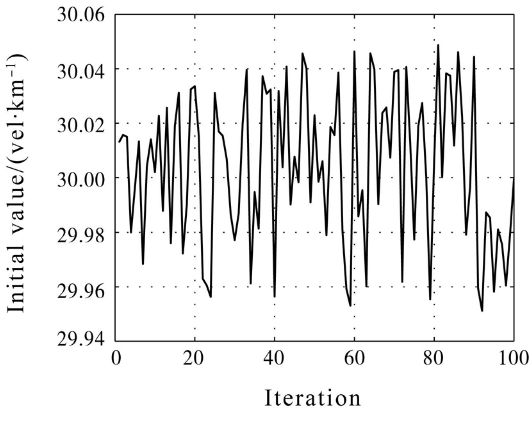

Fig. 4 The profile of random initial value $\rho_7(0)$

The control input of the first iteration is equal to 0. Fig.5 shows the maximum tracking error of the traffic flow density in Section 7. For practical urban freeway traffic control system, from Figs.4–5 we can see that the proposed scheme can overcome the limitations of traditional ILC with respect to identical initial condition, and achieve the perfect tracking.

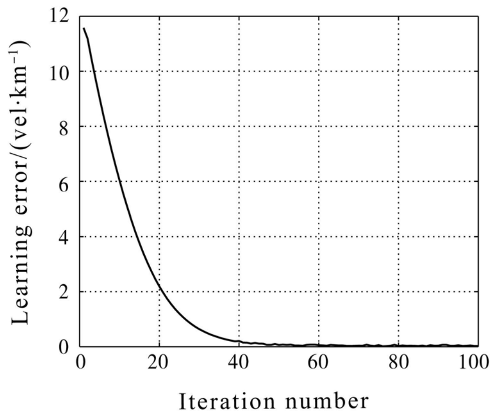

Fig. 5 $\max\limits_{t \in \{0,500\}} e_k(t)$ with the proposed MFAILC

5 Conclusions

In this work, MFAILC has been proposed based on a new PFDL approach in the iteration domain. The monotonic convergence of the iteration maximum error can be guaranteed by theoretical analysis when the initial conditions are randomly varying along the iteration axis. The main feature of the proposed control scheme is that the controller design only depends on the I/O data of the controlled plant, so MFAILC is essentially a data-driven control method. The effectiveness of the proposed MFAILC is verified by the extensive numerical simulations.

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作者简介:

金尚泰 (1976–), 男, 讲师, 研究领域包括无模型自适应控制、数据驱动控制、学习控制等, E-mail: shtjin@bjtu.edu.cn;

侯忠生 (1962–), 男, 教授, 博士生导师, 研究领域包括无模型自适应控制、数据驱动控制、学习控制、智能交通系统、智能信息处理等, E-mail: zhshhou@bjtu.edu.cn;

池荣虎 (1975–), 男, 教授, 研究领域包括无模型自适应控制、数据驱动控制、学习控制等, E-mail: rhchi@163.com;

柳向斌 (1971–), 男, 教授, 博士生导师, 研究领域包括非线性控制、自适应控制、数据驱动控制等, E-mail: xbliu@bjtu.edu.cn.