

## 非线性离散时间系统的最优终端迭代学习控制

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**摘要:** 仅利用系统的终端输出误差而不是整个输出轨迹, 提出了一种最优终端迭代学习控制方法. 控制信号可直接通过终点的误差信息进行更新. 主要创新点在于控制器的设计和分析只利用系统量测的I/O数据而不需要关于系统模型的任何信息, 并可实现沿迭代轴的单调收敛. 在此意义上, 所提出的控制器是数据驱动的无模型控制方法. 严格的数学分析和仿真结果均表明了所提出方法的适用性和有效性.

**关键词:** 终端迭代学习控制; 最优设计; 数据驱动控制; 非线性系统

**中图分类号:** TP273      **文献标识码:** A

## An optimal terminal iterative learning control approach for nonlinear discrete-time systems

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**Abstract:** This paper presents an optimal terminal iterative learning control (TILC) approach by considering only the terminal output tracking error instead of the whole output trajectory tracking error. The control signal is directly updated from the error information of the given final terminal point. The key contributions of the presented optimal terminal iterative learning control (ILC) is that the controller design and analysis only uses the measured I/O data without any modeling information of the plant and the monotonic convergence is guaranteed. In this sense, the proposed controller is a data-driven approach. Both rigorous mathematical analysis and simulation results illustrate the applicability and effectiveness of the proposed approach.

**Key words:** terminal iterative learning control (ILC); optimality-based design; data-driven control; nonlinear systems

### 1 Introduction

There is a new control scenario: i) the system dynamics ends in a finite time interval and repeats, ii) the only available measurement is the terminal state or terminal output, iii) and the ultimate control objective is also the terminal state or terminal output instead of the whole trajectory of the system. For example, in the rapid thermal processing (RTP) of wafer industry<sup>[1]</sup>, the exact measurement of wafer temperature is almost impossible due to the insufficient measurement capability, and the ultimate control objective is to control the deposition thickness (DT) at the end of the RTP cycle.

By virtue of repeatability, iterative learning control (ILC)<sup>[2-9]</sup> is most suitable to improve system control

performance using its learning ability from repetitions. However, the conventional ILC that drives to follow a desired output trajectory in a given time interval through learning iteratively, is not applicable to such a control task because the intermediate measurements of the system state and output are impossible except for the terminal output.

To overcome these problems, terminal iterative learning control (TILC)<sup>[1]</sup> was extended from ILC theory to use the terminal point only at the end of every run. Now terminal ILC is becoming a new research direction of ILC for the requirements of many practical applications, such as chemical vapor deposition<sup>[1]</sup>, thermoforming process<sup>[10]</sup>, and chemical reactor<sup>[11]</sup> in batch

Received 5 May 2012; revised 29 June 2012.

This work is supported by National Science Foundation of China (Nos. 60974040, 60834001, 61120106009), the Research Award Foundation for the Excellent Youth Scientists of Shandong Province of China (No. BS2011DX010), and the Fundamental Research Funds for the Central Universities (No. 2011JBM201).

processes, as well as the point-to-point control in a two-link flexible arm system<sup>[12]</sup> and the station stop control of a train<sup>[13]</sup>. However, in comparison with applications, the existing theoretical results of TILC are too few and mainly focus on P-type learning laws<sup>[1, 10, 12–13]</sup> and optimal learning mechanism<sup>[14]</sup> for linear systems. The exact lower and upper bounds of coefficient matrices in linear systems must be known a priori to select the learning gain of P-type TILC so as to satisfy the convergence conditions.

Although optimization based TILC<sup>[14]</sup> can guarantee monotonic tracking error convergence and overcome the poor transient performance along iteration axis compared with P-type learning control laws<sup>[1, 10, 12–13]</sup>, a major limitation of norm optimal terminal ILC is the requirement of a perfect model to calculate the learning gain via solving a Riccati equation. Similar to the existing norm optimal ILC approaches<sup>[15–19]</sup>, when the model is inaccurate, its monotonic convergence is no longer guaranteed and learning transients including large, rapid growth of the error or even instability will occur. As a direct result, the normal optimal TILC is lack of flexibility regarding modifications and expansions of the controlled plant in practice due to its dependence on the accurate known linear model of the system dynamics.

For a practical nonlinear plant, however, the exact linear model of the plant is often difficult to develop. Sometimes, it is impossible. Furthermore, even though a linear model is constructed, the optimal learning control system would inherit a born instability caused by the unmodeled dynamics, which always exists in the modeling process. For example, a common approach optimal leaning control design for nonlinear systems is to develop a linearization model using Taylor series expansion, while the neglected higher order terms will affect the exact values of systems Jacobian matrices to calculate the optimal control gains. Recently, [20] proposed a neural network model based batch-to-batch optimal TILC strategy for nonlinear systems, but the selection of a proper neural network requires some efforts in practice.

It is worth pointing out that in many real processes, a most prominent feature is the presence of vast volume of data although there is lack of an effective process physical model that can support control, fault diagnosis, scheduling, and decision making<sup>[21–23]</sup>. This motivates us to explore a data-driven optimal terminal ILC method, which does not require any model information but the input and output measurements of the controlled plants. Then we can enjoy not only extra good properties of the norm optimal terminal ILC, but also the little requirements on the system dynamic model.

In this work, a new data-driven design for optimal terminal ILC is developed for nonlinear systems. The presented approach consists of an iteration-recursive optimal learning law of control input and an iterative optimal estimating law of partial derivative (PD). The distinct features of the presented approach are as follows:

a) It is proposed for a class of completely unknown nonlinear non-affine systems and the only priori is the existence of the boundary of system partial derivative with respect to control input.

b) A linear incremental dynamical mapping relationship of input-output is developed between two consecutive iterations. It is quite different from the input-output relationship based on lifting technology and super vector transformation<sup>[14]</sup> since the latter is a static mapping and cannot reflect the system dynamics realtimely.

c) The learning gain of the optimal control law is calculated by using the iterative estimation values of PD parameters not by solving a static Riccati equation like the typical optimal approaches<sup>[14–19]</sup>, and thus able to be updated iteratively and flexible to the modifications and expansions of the controlled plant.

d) The proposed approach is a data-driven model-free control strategy, since the controller design and analysis requires only the measurement I/O data without using any model information of the plant.

The rest of this paper is organized as follows. Section 2 gives the problem formulation. The optimal TILC design is developed in Section 3. Section 4 shows the stability and convergence of the TILC system with rigorous analysis. Some simulation results are provided in Section 5. Finally, some conclusions are given in Section 6.

## 2 Problem formulation

To clearly demonstrate the main idea, we first consider a discrete-time single-input-single-output (SISO) nonlinear discrete-time as follows:

$$y_k(t+1) = f(y_k(t), u_k), \quad (1)$$

where  $t = 0, 1, \dots, N$  is the sampling time index,  $N$  is the finite time interval of the run-to-run system;  $k$  indicates the system repetition number;  $y_k(t) \in \mathbb{R}$  is the system output, where only  $y_k(N)$  is measurable at the end of every run;  $u_k \in \mathbb{R}$  denotes the system input, which is time-invariant at all sampling time in the same run;  $f$  is an unknown function and continuously differentiable.

Over each trial, the relationship between the input and output time-series can be expressed by the following algebraic functions:

$$\left\{ \begin{aligned} y_k(1) &= f(y_k(0), u_k) = g^1(y_k(0), u_k), \\ y_k(2) &= f(y_k(1), u_k) = f(g^1(y_k(0), u_k), u_k) = \\ &\quad g^2(y_k(0), u_k), \\ y_k(3) &= f(y_k(2), u_k) = f(g^2(y_k(0), u_k), u_k) = \\ &\quad g^3(y_k(0), u_k), \\ &\quad \vdots \\ y_k(N) &= f(y_k(N-1), u_k) = \\ &\quad f(g^{N-1}(y_k(0), u_k), u_k) = \\ &\quad g^N(y_k(0), u_k), \end{aligned} \right. \quad (2)$$

where  $y_k(0)$  is the initial value of system (1),  $g^1(\cdot, \cdot), \dots, g^N(\cdot, \cdot)$  are the corresponding nonlinear functions and differentiable to all the arguments.

To reveal the dynamical relationship of system (1) among iterations, one can explore the difference of  $\Delta y_k(N)$  along the iteration axis, i.e.,

$$\begin{aligned} \Delta y_k(N) &= y_k(N) - y_{k-1}(N) = \\ &g^N(y_k(0), u_k) - g^N(y_{k-1}(0), u_{k-1}) = \\ &g^N(y_k(0), u_k) - g^N(y_k(0) - \Delta y_k(0), u_k - \Delta u_k), \end{aligned} \quad (3)$$

where  $\Delta y_k(0) = y_k(0) - y_{k-1}(0)$ ,  $\Delta u_k = u_k - u_{k-1}$ . Using the mean value theorem,

$$\begin{aligned} \Delta y_k(N) &= g^N(y_k(0), u_k) - [g^N(y_k(0), u_k) - \\ &g_y^N(\xi_k)\Delta y_k(0) - g_u^N(\zeta_k)\Delta u_k = \\ &g_y^N(\xi_k)\Delta y_k(0) + g_u^N(\zeta_k)\Delta u_k, \end{aligned} \quad (4)$$

where  $g_y^N = \frac{\partial g}{\partial y}$ ,  $g_u^N = \frac{\partial g}{\partial u}$  and  $\xi_k \in [y_k(0) - |\Delta y_k(0)|, y_k(0) + |\Delta y_k(0)|]$ ,  $\zeta_k \in [u_k - |\Delta u_k|, u_k + |\Delta u_k|]$ .

To restrict our discussion, the nonlinear system (1) is assumed to satisfy the following assumptions.

**Assumption 1** System (1) is completely controllable.

**Assumption 2** The initial value  $y_k(0)$  is assumed identical for every iteration  $k$ , i.e.,  $y_k(0) = y_{k-1}(0) = c$  with  $c$  being a constant.

**Assumption 3**  $g_u^N$  has lower and upper bounds, both are of the same sign and strictly nonzero, i.e., if assume  $\alpha_1$  the lower bound and  $\alpha_2$  the upper bound, then either  $0 < \alpha_1 < \alpha_2$  or  $0 > \alpha_2 > \alpha_1$ . Without loss of generality, in this paper we assume  $0 < \alpha_1 < \alpha_2$ .

**Remark 1** Assumptions 1–3 are quite standard as these are widely considered to design a learning control algorithm<sup>[1–20]</sup>. In addition, we just need the existence of the lower and upper bounds in Assumption 3 without requiring their exact values.

**Remark 2** It is worth pointing out that  $g_u^N$  is the equivalent process gain like that in linear case. However, in the non-affine case the process gain  $g_u^N$  is depending on the control input  $u$ . Thus it is also necessary to limit  $u$ , especially when  $g_u^N$  turns out to be a radially

unbounded function of  $u$ , i.e.,  $\lim_{|u| \rightarrow \infty} |g_u^N| \rightarrow \infty$ . In such circumstance we have to limit  $u$  to a compact set  $S_u$ . By virtue of the continuous differentiability of  $g_u^N$ ,  $g_u^N$  is bounded on  $S_u$ <sup>[24]</sup>. The boundedness of control input  $u$  is guaranteed in the following discussion.

**Remark 3** Note that it is essential that  $g_u^N \neq 0$  for non-affine cases because the singularity yields a zero process gain, hence the system is uncontrollable at the singular points.

In terms of Assumption 2, Eq.(4) becomes

$$\Delta y_k(N) = g_u^N(\zeta_k)\Delta u_k = \theta_k\Delta u_k, \quad (5)$$

where  $\theta_k = g_u^N(\zeta_k)$ . According to Assumption 3,  $0 < \alpha_1 < \theta_k < \alpha_2$  holds for all iterations  $k$ .

Eq.(5) can be rewritten as

$$y_k(N) = y_{k-1}(N) + \theta_k\Delta u_k. \quad (6)$$

The control target of TILC is to track a given desired output  $y_d(N)$  at the single terminal point during system operation by generating an optimal control signal  $u_d$  through trials.

### 3 Optimal terminal ILC design

Define tracking error  $e_k(N) = y_d - y_k(N)$ . Consider an index function of the control input as follows:

$$J(u_k) = |e_k(N)|^2 + \lambda|u_k - u_{k-1}|^2, \quad (7)$$

where  $\lambda > 0$  is a weighting factor.

According to Eqs.(6)–(7) can be rewritten as

$$\begin{aligned} J(u_k) &= |y_d - y_{k-1}(N) - \theta_k(u_k - u_{k-1})|^2 + \\ &\lambda|u_k - u_{k-1}|^2 = \\ &|e_{k-1}(N) - \theta_k(u_k - u_{k-1})|^2 + \\ &\lambda|u_k - u_{k-1}|^2. \end{aligned} \quad (8)$$

By the optimal condition, we have

$$u_k = u_{k-1} + \frac{\rho\theta_k}{\lambda + \theta_k^2}e_{k-1}(N), \quad (9)$$

where  $\rho$  is a positive constant, added as a step-size constant series to make the generality of algorithm (9), and will be used in the following analytical convergence proof.

Since  $\theta_k$  is unknown, we give an estimate algorithm for it. Consider a new index function as

$$\begin{aligned} J(\hat{\theta}_k) &= |\Delta y_{k-1}(N) - \hat{\theta}_k(u_{k-1} - u_{k-2})|^2 + \\ &\mu|\hat{\theta}_k - \hat{\theta}_{k-1}|^2. \end{aligned} \quad (10)$$

According to Eq.(6), we can rewrite Eq.(10) as

$$\begin{aligned} J(\hat{\theta}_k) &= |y_{k-1}(N) - (y_{k-2}(N) - \hat{\theta}_{k-1}\Delta u_{k-1}) - \\ &(\hat{\theta}_k - \hat{\theta}_{k-1})\Delta u_{k-1}|^2 + \mu|\hat{\theta}_k - \hat{\theta}_{k-1}|^2. \end{aligned} \quad (11)$$

The estimate algorithm is shown as follows in term of the optimal condition:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\eta(\Delta y_{k-1}(N) - \hat{\theta}_{k-1}\Delta u_{k-1})\Delta u_{k-1}}{\mu + \Delta u_{k-1}^2}, \quad (12)$$

where  $0 < \eta < 2$  is a positive constant, added as a step-size constant series to make the generality of Eq.(12), and  $\hat{\theta}_0$  is selected as  $\hat{\theta}_0 > \varepsilon > 0$ .

**Remark 4** Without loss of generality, the initial value  $\hat{\theta}_0$  in Eq.(12) can be chosen as zero if there is not any information about  $\theta_k$  available.

Correspondingly, the control law (9) becomes

$$u_k = u_{k-1} + \frac{\rho \hat{\theta}_k}{\lambda + \hat{\theta}_k^2} e_{k-1}(N). \quad (13)$$

In order to make the parameter estimation algorithm (12) have a strong tracking ability, we present a reset algorithm as follows:

$$\hat{\theta}_k = \hat{\theta}_0, \text{ if } |\hat{\theta}_k| \leq \varepsilon \text{ or } \text{sgn } \hat{\theta}_k \neq \text{sgn } \hat{\theta}_0. \quad (14)$$

**Remark 5** The optimal terminal ILC (12)–(14) is designed only by the system input and output measurement data. In other word, there is neither the explicit model dynamics nor the structural information of the plant used in the presented terminal ILC.

**Remark 6** Compared to other optimal TILC or ILC methods, the presented optimal TILC law is non-linear and the learning gain can be iteratively updated by Eq.(12).

#### 4 Convergence analysis

The validity of the above presented optimal terminal ILC (12)–(14) is verified by the following theorem.

**Theorem 1** For the general nonlinear system (1) under Assumptions 1–3, applying the presented optimal TILC (12)–(14), we can guarantee that:

- a) The partial derivative parameter estimation value  $\hat{\theta}_k$  is bounded for all iterations  $k$ ;
- b) The terminal tracking error  $e_k(N)$  converges to zero iteratively and monotonically as iteration number  $k$  approaches to infinity, i.e.,  $\lim_{k \rightarrow \infty} e_k(N) = 0$ ;
- c) The system terminal output  $y_k(N)$  and the constant control input  $u_k$  are bounded for all iterations.

**Proof** There are three parts for the theorem proof, as shown in the following details:

- i) The boundedness of estimation value  $\hat{\theta}_k$ .

Define  $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$ . Subtracting  $\theta_k$  from both sides of Eq.(12),

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + (\theta_k - \theta_{k-1}) - \frac{\eta(\Delta y_{k-1}(N) - \hat{\theta}_{k-1} \Delta u_{k-1}) \Delta u_{k-1}}{\mu + \Delta u_{k-1}^2}. \quad (15)$$

According to Eq.(6), Eq.(15) becomes

$$\begin{aligned} \tilde{\theta}_k &= \tilde{\theta}_{k-1} + (\theta_k - \theta_{k-1}) - \\ &\frac{\eta(\theta_{k-1} \Delta u_{k-1} - \hat{\theta}_{k-1} \Delta u_{k-1}) \Delta u_{k-1}}{\mu + \Delta u_{k-1}^2} = \\ \tilde{\theta}_{k-1} &- \frac{\eta \Delta u_{k-1}^2 \tilde{\theta}_{k-1}}{\mu + \Delta u_{k-1}^2} + (\theta_k - \theta_{k-1}) = \end{aligned}$$

$$\left(1 - \frac{\eta \Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}\right) \tilde{\theta}_{k-1} + (\theta_k - \theta_{k-1}). \quad (16)$$

Taking norm of both sides of Eq.(16),

$$\begin{aligned} |\tilde{\theta}_k| &\leq \left|1 - \frac{\eta \Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}\right| |\tilde{\theta}_{k-1}| + \\ &|\theta_k - \theta_{k-1}|. \quad (17) \end{aligned}$$

Since  $0 < \eta < 2$  and  $\mu > 0$ , then there exists a constant  $0 < d_1 < 1$  such that

$$0 < \left|1 - \frac{\eta \Delta u_{k-1}^2}{\mu + \Delta u_{k-1}^2}\right| \leq d_1 < 1. \quad (18)$$

According to Assumption 3,  $0 < \alpha_1 < \theta_k < \alpha_2$ . From Eqs.(17)–(18), one obtains

$$|\tilde{\theta}_k| \leq d_1 |\tilde{\theta}_{k-1}| + \alpha_2 - \alpha_1 \leq \dots \leq d_1^k |\tilde{\theta}_0| + \frac{\alpha_2 - \alpha_1}{1 - d_1}. \quad (19)$$

Since the initial estimate error  $\tilde{\theta}_0$  is bounded, then conclusion a) of Theorem 1 directly follows from Eq.(16), i.e., both of  $\tilde{\theta}_k$  and  $\hat{\theta}_k$  are bounded.

- ii) The tracking error convergence.

From Eq.(6) and control law (13), we have

$$e_k(N) = y_d - y_{k-1}(N) - \theta_k(u_k - u_{k-1}) =$$

$$\begin{aligned} e_{k-1}(N) - \frac{\rho \theta_k \hat{\theta}_k}{\lambda + \hat{\theta}_k^2} e_{k-1}(N) = \\ \left(1 - \frac{\rho \theta_k \hat{\theta}_k}{\lambda + \hat{\theta}_k^2}\right) e_{k-1}(N). \quad (20) \end{aligned}$$

According to the reset algorithm (14), apparently the sign of  $\theta_k$  and  $\hat{\theta}_k$  is identical, hence  $\theta_k \hat{\theta}_k$  is positive as a direct result. Furthermore, the boundedness of  $\theta_k$  and  $\hat{\theta}_k$  has been shown above, so by properly selecting  $\rho$  and  $\lambda$ , one can assure that

$$0 < \left|1 - \frac{\rho \theta_k \hat{\theta}_k}{\lambda + \hat{\theta}_k^2}\right| \leq d_2 < 1. \quad (21)$$

Thus Eq.(20) yields

$$\begin{aligned} |e_k(N)| &= \left|1 - \frac{\rho \theta_k \hat{\theta}_k}{\lambda + \hat{\theta}_k^2}\right| |e_{k-1}(N)| \leq \\ d_2 |e_{k-1}(N)| &\leq \dots \leq d_2^k |e_0(N)|. \quad (22) \end{aligned}$$

Apparently, when iteration  $k$  approaches infinity, the terminal tracking error goes to zero iteratively and monotonically.

- iii) The boundedness of system output and control input.

Since the target terminal point  $y_d(N)$  is given bounded and the tracking error  $e_k(N)$  has been shown convergent and bounded for all iterations  $k$ , then the boundedness of  $y_k(N)$  is obvious.

The boundedness of  $\hat{\theta}_k$  yields

$$\left|\frac{\rho \hat{\theta}_k}{\lambda + \hat{\theta}_k^2}\right| \leq M, \quad (23)$$

where  $M$  is a positive scalar.

It is clear that

$$u_k = u_k - u_{k-1} + u_{k-1} - \dots + u_1 - u_0 + u_0 = u_0 + \sum_{j=1}^k \Delta u_j. \quad (24)$$

In terms of Eqs.(13)(22)–(24), using the Schwarz inequality, we have

$$\begin{aligned} |u_k| &\leq |u_0| + \sum_{j=1}^k |\Delta u_j| \leq \\ &|u_0| + \sum_{j=1}^k \left| \frac{\rho \hat{\theta}_k}{\lambda + \hat{\theta}_k^2} \right| |e_{j-1}(N)| \leq \\ &|u_0| + M \sum_{j=1}^k d^{j-1} |e_0(N)| \leq \\ &|u_0| + M \frac{d}{1-d} |e_0(N)|. \end{aligned} \quad (25)$$

Thus, the control input is bounded for all iterations  $k = 1, 2, \dots$ . And the conclusion c) of Theorem 1 is obtained.

### 5 Illustrative examples

Consider an ethanol fermentation process<sup>[25]</sup>, whose mechanistic model in the form of differential algebraic equations (DAE) is described as follows<sup>[26]</sup>. It should be noted that the mathematical model is assumed to be unavailable, and just serve as the I/O data generator for the systems to be controlled, no any information of them will be included in the controller design.

$$\begin{cases} \frac{dx_1}{dt} = Cx_1 - \frac{x_1}{x_4}u, \\ \frac{dx_2}{dt} = -10Cx_1 - \frac{(150 - x_2)}{x_4}u, \\ \frac{dx_3}{dt} = Dx_1 - \frac{x_3}{x_4}u, \\ \frac{dx_4}{dt} = u, \\ y = x_3, \end{cases} \quad (26)$$

where  $C = \frac{0.408x_2}{(1 + x_3/16)(0.22 + x_2)}$  and  $D = \frac{x_2}{(1 + x_3/71.5)(0.44 + x_2)}$ ;  $x_1$  is the cell mass concentration;  $x_2$  is the substrate concentration;  $x_3$  is the product concentration; and  $x_4$  is the liquid volume of the reactor.  $x_4$  is limited by the 200 L vessel size. The initial condition is specified as  $x(0) = [1 \ 150 \ 0 \ 10]^T$ . The batch length  $t_f$  is fixed to be 63.00 hours and divided into  $N = 10$  equal stages (i.e., sampling time  $h = t_f/N = 6.3$ ). The feed rate into the reactor  $u$  is used for control and constrained by  $0 \leq u \leq 12(1/h)$ . There is no outflow, so the feed rate must be chosen so that the batch volume does not exceed the physical volume of the reactor.

The desired output  $y_d(N) = 103.53$ , was selected

from the literature<sup>[25]</sup>. The controller parameters of the presented optimal terminal ILC (12)–(14) were chosen as  $\rho = 0.1, \eta = 1, \lambda = 1, \mu = 1$ . Fig.1 shows the convergence of terminal error with respect to the iterations. The horizon is the iteration number and the vertical axis is the absolute values of terminal tracking error. Fig.2 shows the control input profile with respect to the iterations.

It is obvious that the presented approach has the converging properties. The terminal tracking error converges to zero iteratively, and the control input signal is bounded for all iterations and converges to its optimal value with iteration number increasing.

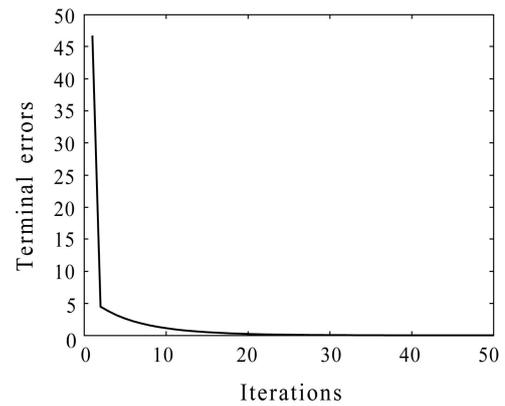


Fig. 1 The terminal tracking error profile

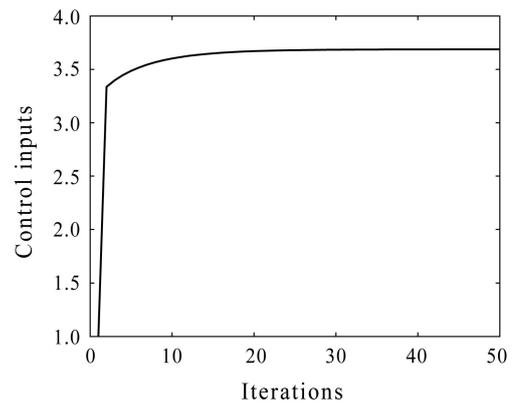


Fig. 2 The control signal profile

### 6 Conclusions

The requirement for data-driven control approaches arises in more and more practical applications. This paper shows how it may be tackled in the optimal TILC design. The presented data-driven optimal design scheme provided a general framework of TILC for nonlinear systems. The controller design and analysis only depends on the real-time measured I/O data of the plant without requiring any other model information. Rigorous mathematical analysis and extensive simulations are developed to illustrate the efficiency of the proposed approach.

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