第 27 卷第 12 期
2010 年 12 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 27 No. 12
Dec. 2010

Article ID: 1000−8152(2010)12−1757−09

An output delay approach to sensor fault detection for non-uniformly sampled-data systems

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Abstract: In this paper, the problem of sensor fault detection in general non-uniformly sampled-data systems is investigated. First, the output delay approach is used to model non-uniformly sampled-data systems as continuous ones with time-varying delay output. Then, based on the input-output approach, a fault detection filter is designed to guarantee not only the effects of continuous-time process noise and discrete-time measurement noise on residual to satisfy a prescribed H-infinity performance, but also the l-two gain from sensor fault to residual to be greater than a prescribed value. The existence condition of such a fault detection filter is given in terms of matrix inequalities(MIs). Furthermore, an iterative algorithm which transform MIs into the solvable linear MIs(LMIs) is proposed to make a tradeoff between the noise robustness and fault sensitivity. Finally, simulation results of a certain type of flight control systems are presented to show the validity of the proposed method.

Key words: non-uniformly sampled-data systems; fault detection filters; output delay approach; iterative algorithm; sensor fault

CLC number: TP273 **Document code:** A

基于输出时滞方法的非均匀采样数据系统传感器故障检测

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摘要: 研究一般非均匀采样数据系统鲁棒传感器故障检测设计问题. 首先, 基于输出时滞方法将非均匀采样数据系统转换成具有时变时滞输出的连续系统; 然后, 选择故障检测滤波器作为残差产生器, 并将故障检测设计问题描述成一个多目标优化问题, 即连续时间过程噪声和离散时间测量噪声对残差信号的H_∞范数小于一个给定值, 同时传感器故障对残差信号的l_2增益大于一个给定值, 基于输入输出方法以矩阵不等式的形式给出该多目标优化问题有解的充分条件; 进一步的, 提出一个迭代算法来权衡噪声鲁棒性与故障灵敏度, 并将矩阵不等式转换成可解的线性矩阵不等式. 最后, 对某型飞控系统的仿真实验验证了所提方法的有效性.

关键词: 非均匀采样数据系统; 故障检测滤波器; 输出时滞方法; 迭代算法; 传感器故障

1 Introduction

Modern complex control systems are widely exposed to various faults which may drastically change the system behavior, resulting in performance degradation, instability and even total breakdown. In order to maintain the system safety and reliability, faults should be promptly detected and identified so that appropriate remedies can be applied. During the last three decades, fault detection and isolation(FDI) algorithms with their applications have attracted remarkable attention. Fruitful results can be found in several excellent books and survey papers and reference there in [1∼4].

On the other hand, sampled-data systems are extensively used and accepted in industry because of numerous advantages of digital technology. In these types of systems the plant operates in continuous time while the system outputs are sampled, yielding discrete-time signals. Sampled-data systems are thus hybrid systems, involving both continuous-time and discrete-time signals[5]. The traditional FDI approaches to sampled-data systems are indirectly accomplished by the existing continuous-time and discrete-time FDI technolo-

Received data: 2010−05−07; Revised data: 2010−10−20.
Foundation item: Supported by Key Program of National Nature Science Foundation of China(60934009).

gies[6,7]. However, these indirect approaches ignore what happens between the sampling instants (the intersample behavior), so approximations are involved, resulting in unsatisfactory performances of FDI systems[8]. Similar to the sampled-data controller design problem, the intersample behavior of system is the focus of direct design methods of sampled-data FDI. Based on the continuous lifting technique which can effectively capture the intersample behavior, the parity-based residual generator, optimal fault detection filter and optimal diagnostic observer are developed in [9～11] respectively. Furthermore, by showing that norms of a sampled system are equal to the corresponding norms of a certain discrete time system, a new discretization method called norm invariant transformation is proposed in [12] to convert the norm-based sampled-data fault detection problem into an equivalent discrete time one. However, all above direct design approaches based on continuous lifting technique require the sampled-data systems under consideration to be strictly proper and are thus not applicable for the sampled-data systems with measurement noise or sensor fault. The hybrid systems approach is another effective way to deal with the intersample behavior. The optimal fault detection filter for general sampled-data systems is developed in [13] based on the co-inner-outer factorization technique of linear jump systems. By formulating the sampled-data FDI problem as hybrid system filtering one, the actuator fault detection and sensor fault estimation problems are investigated in [14,15] respectively. However, the results in [13～15] are given in terms of Riccati differential equations with jumps which are difficult to be sloved. Therefore, an LMI solution of actuator fault detection problem of general sampled-data systems are given in [16], which verifies the feasibility of direct method based on hybrid system approach.

All above direct design methods based on continuous lifting techniques or hybrid system approaches usually assume that each process variable is sampled at a constant rate. The sampling rates of different variables may be equal and synchronous (i.e., single-rate systems) or different and asynchronous (i.e., multirate rate)[17,18]. However, in many practical industry processes or complex control systems, the output is sampled at non-uniformly spaced time instants due to different reasons, including the delays in sensors, networks and laboratory analysis. It is also well known that introducing non-uniform sampling can provide a better tradeoff between performance and implementation cost and can achieve objectives that can not achieved by uniform sampling[19,20]. By using subspace identification method, the parity-based and Kalman filter based FDI methodologies for non-uniformly sampled-data systems are developed in [21] and [22] respectively. However it is assumed that sampling, although non-uniform, follows a periodic pattern. In other words, the sampling instants are non-uniformly distributed in a window of time, and this window is periodically repeated. This periodicity assumption allows the use of discrete-time lifting technique to convert the non-uniformly sampled-data system into linear time invariant(LTI) systems, but it restricts the practical applications of the proposed method. By using continuous lifting technique, the parity-based residual generator for general non-uniformly sampled-data systems is proposed in [23]. However, the obtained time-varying residual generator needs for calculation at every sampling instant and thus has a large computation cost. Furthermore, as mentioned above, the use of continuous lifting technique requires the system under consideration to be strictly proper, which means no measurement noise and sensor fault exist. To the best of authors' knowledge, the problem of sensor fault detection in general non-uniformly sampled-data systems has not yet been fully investigated. The difficulties lie in several aspects, such as how to convert the non-uniformly sampled-data system into system easily to deal with (e.g., in [21,22], the non-uniformly sampled data system is converted into LTI system), and how to make a good tradeoff between the fault sensitivity and noise robustness.

The design of sensor fault detection filters for general non-uniformly sampled-data systems will be studied in this paper. Only a conventional restriction on sampling is that the interval between sequel sampling instants does not exceed a given bound. Inspired by the time delay approach which has been proposed in [24,25] for sampled-data stabilization and H_∞ control, the output delay approach will be employed to convert non-uniformly sampled-data systems into continuous-time ones with time-varying delay output. More recently, the fault diagnosis and fault tolerant control for system with delayed output have already been investigated in [26,27], by use of an exosystem and the delay free transform, the fault diagnosis problem for system with delayed outputs has been converted into a state estimation problem for a delay free system. However, the dynamic characters of faults are assumed to be known and only constant delay is considered, which may place some restrictions on realistic FDI. In this paper, the proposed method does not need any assumption on faults and the time-varying delay is well dealt with by the input-output approach. An iterative algorithm is finally developed to make a good tradeoff between noise robustness and fault sensitivity.

2 Problem formulation

Consider the following sampled-data system

$$\begin{cases} \dot{x}(t)=Ax(t)+B_{\rm u}u(t)+B_{\rm w}w(t), x(0)=x_0, \\ y(t_k) = Cx(t_k) + D_{\rm v}v(t_k) + D_{\rm f}f_{\rm s}(t_k), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^{n_{\rm u}}$ is the control input, $w(t) \in \mathbb{R}^{n_{\rm w}}$ is the continuous-time process noise belongs to $\mathrm{L}_2[0,\infty)$ which stands for the space of square integrable vector functions over $[0,\infty)$, $y(t_k) \in \mathbb{R}^m$ is the sampled output at time instant t_k, $v(t_k) \in \mathbb{R}^{n_{\rm v}}$ is the discrete-time measurement noise belongs to $\mathrm{l}_2[0,\infty)$ which is the space of square summable vector sequences over $[0,\infty)$, $f_{\rm s}(t_k) \in \mathbb{R}^{n_{\rm f}}$ is the sensor fault to be detected. $A, B_{\rm u}, B_{\rm w}, C, D_{\rm v}, D_{\rm f}$ are known real constant matrices with approximate dimensions, and the initial x_0 is also known. The sampling is non-uniform or even uncertain and the discrete sampling instants are

$$0 = t_0 < t_1 < \cdots < t_k < \cdots, \lim_{k\to\infty} t_k = \infty.$$

We further assume that the maximum sampling interval is not greater than h, i.e.,

$$t_{k+1} - t_k \leqslant h, \forall k \geqslant 0. \tag{2}$$

Between the sampling instants, the fault detection filter can only use the control input $u(t)$ and the sampled output $y(t_k)$ at previous sampling instant to detect the fault, so the sampled output equation may be represented as a continuous time one with time-varying delay as follows:

$$\begin{aligned} y(t) = &Cx(t-\tau(t)) + D_{\rm v}v(t-\tau(t)) + \\ &D_{\rm f}f_{\rm s}(t-\tau(t)), \end{aligned} \tag{3}$$

where $\tau(t) = t - t_k, t_k \leqslant t < t_{k+1}$. Meanwhile, the following fault detection filter is constructed

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B_{\rm u}u(t) + L(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t-\tau(t)), r(t) = H(y(t) - \hat{y}(t)), \end{cases} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state estimation, $\hat{x}(0) = x_0$, $\hat{y}(t) \in \mathbb{R}^m$ is the output estimation, $r(t) \in \mathbb{R}^{n_{\rm r}}$ is the residual. $L \in \mathbb{R}^{n\times m}$ is the filter gain matrix to be designed, $H \in \mathbb{R}^{n_{\rm r}\times m}$ is a suitable post weighting matrix designed to assure isolability properties. Define the filter error $e(t) = x(t) - \hat{x}(t)$, then the error dynamics is described by

$$\begin{cases} \dot{e}(t)=Ae(t) - LCe(t-\tau(t)) + B_{\rm w}w(t)- \\ \qquad LD_{\rm v}v(t-\tau(t)) - LD_{\rm f}f_{\rm s}(t-\tau(t)), \\ r(t)=HCe(t-\tau(t)) + HD_{\rm v}v(t-\tau(t))+ \\ \qquad HD_{\rm f}f_{\rm s}(t-\tau(t)), \end{cases} \tag{5}$$

where $r(t)$ is represented in continuous-time case, while it is, in essence, the discrete-time non-uniformly sampled signal. To effectively detect the fault, the fault detection filter needs to be strong robust against the noise and simultaneously high sensitivity to the fault. Therefore, based on the above motivation, we now formulate the design problem as follows:

Sensor fault detection filter problem: Under the non-uniformly sampled-data system (1) and the fault detection filter (4), determining the filter gain matrix L such that

1) Filter (4) is asymptotically convergent for all $w = 0, v = 0$ and $f_{\rm s} = 0$.

2) For a given scalar $\gamma > 0$, the following inequality holds for any non-zero w, v

$$J_{\rm r} < 0, \tag{6}$$

where

$$\begin{aligned} J_{\rm r} = &\sum_{k=0}^{\infty} (t_{k+1} - t_k) r^{\rm T}(t_k) r(t_k) - \\ &\int_0^{\infty} \gamma^2 w^{\rm T}(t) w(t) {\rm d}t - \\ &\gamma^2 \sum_{k=0}^{\infty} (t_{k+1} - t_k) v^{\rm T}(t_k) v(t_k). \end{aligned} \tag{7}$$

3) For a given scalar $\beta > 0$ and the residual evaluation time T, the following inequality holds for any non-zero $f_{\rm s}$

$$\sum_{k=0}^{T} r^{\rm T}(t_k) r(t_k) > \beta^2 \sum_{k=0}^{T} f_{\rm s}^{\rm T}(t_k) f_{\rm s}(t_k). \tag{8}$$

4) γ/β is as small as possible.

Remark 1 Equation (7) is the noise robustness performance index that measures the influence of residual to continuous-time process noise and discrete-time measurement noise. It is evident that the smaller γ is, the less sensitive the residual to noises becomes. Compared with the conventional performance index[28], the noise performance index (7) has some physical sense since it takes the sampling rate into account and thus can approximate the energies of measurement noise and residual. Equation (8) is the fault sensitivity condition. The larger β is, the more sensitive the residual to fault becomes. This class of fault sensitivity performance index has been extensively used in the fault detection for fuzzy systems, Markovian jump systems and so on[29,30]. Note that the length of time window in (8) is finite (T instead of ∞) since in practice it is desired that the fault will be detected as early as possible, an evaluation of residual signal over the whole time range makes less sense. The ratio γ/β indicates a tradeoff between noise robustness and fault sensitivity, the smaller γ/β is, the better the fault detection filter will be.

3 Main results

3.1 Noise robustness analysis

In this section, the input-output approach which has been widely used in the analysis and synthesis for time-delay systems is applied to develop a sufficient condition that guarantees the filter error dynamics (5) is asymptotically stable and satisfies the noise robustness constraint (6). We first represent the error dynamics (5) in the following form:

$$\begin{cases} \dot{e}(t) = \\ (A-LC)e(t) - LC\int_t^{t-\tau(t)} \dot{e}(s)\mathrm{d}s + \\ B_{\mathrm{w}}w(t) - LD_{\mathrm{v}}v(t-\tau(t)) - LD_{\mathrm{f}}f_{\mathrm{s}}(t-\tau(t)), \\ r(t) = \\ HCe(t) + HC\int_t^{t-\tau(t)} \dot{e}(s)\mathrm{d}s + \\ HD_{\mathrm{v}}v(t-\tau(t)) + HD_{\mathrm{f}}f_{\mathrm{s}}(t-\tau(t)). \end{cases} \tag{9}$$

Regarding the integral term in (9) as the additional output of system, we can obtain the following forward system

$$\begin{cases} \dot{e}(t) = (A-LC)e(t) - hLCu_1(t) + B_{\mathrm{w}}w(t) - \\ \qquad LD_{\mathrm{v}}v(t-\tau(t)) - LD_{\mathrm{f}}f_{\mathrm{s}}(t-\tau(t)), \\ r(t) = HCe(t) + hHCu_1(t) + HD_{\mathrm{v}}v(t-\tau(t)) + \\ \qquad HD_{\mathrm{f}}f_{\mathrm{s}}(t-\tau(t)), z(t) = \dot{e}(t), \end{cases} \tag{10}$$

where $z(t)$ is the additional output satisfying $z(t) = 0,\ \forall t < 0$. $u_1(t)$ is the feedback input, i.e.,

$$u_1 = \Delta z \Leftrightarrow u_1(t) = -1/h\int_{t-\tau(t)}^{t} z(s)\mathrm{d}s, \tag{11}$$

where Δ is an operator mapping z to u_1.

Lemma 1 For any matrix $R > 0$, the following inequality holds

$$\int_0^T u_1^{\mathrm{T}}(t)Ru_1(t)\mathrm{d}t \leqslant \int_0^T z^{\mathrm{T}}(t)Rz(t)\mathrm{d}t. \tag{12}$$

Proof By applying Jensen's inequality and changing the order of integral, (12) is easily obtained.

With the aid of Lemma 1, we are now in position to give main results in the subsection.

Theorem 1 Under the non-uniformly sampled-data system (1) and the fault detection filter (4) and assume $f_{\mathrm{s}} = 0$ (fault-free), for a given scalar $\gamma > 0$, if there exist matrices $P_1 > 0$, P_2, P_3, $R > 0$, L that satisfy the following MI:

$$\begin{bmatrix} \Pi_1 & \Pi_2 & -hP_2^{\mathrm{T}}LC & P_2^{\mathrm{T}}B_{\mathrm{w}} & -P_2^{\mathrm{T}}LD_{\mathrm{v}} & C^{\mathrm{T}}H^{\mathrm{T}} \\ * & \Pi_3 & -hP_3^{\mathrm{T}}LC & P_3^{\mathrm{T}}B_{\mathrm{w}} & -P_3^{\mathrm{T}}LD_{\mathrm{v}} & 0 \\ * & * & -hR & 0 & 0 & hC^{\mathrm{T}}H^{\mathrm{T}} \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\gamma^2 I & D_{\mathrm{v}}^{\mathrm{T}}H^{\mathrm{T}} \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{13}$$

where

$$\Pi_1 = P_2^{\mathrm{T}}(A-LC) + (A-LC)^{\mathrm{T}}P_2,$$
$$\Pi_2 = P_1 - P_2^{\mathrm{T}} + (A-LC)^{\mathrm{T}}P_3,$$
$$\Pi_3 = -P_3^{\mathrm{T}} - P_3 + hR.$$

Then the error dynamics (5) is asymptotically stable and the noise robustness constraint (6) is guaranteed.

Proof We first show the stability of (5). Let $v(t_k) = 0$, $w(t) = 0$ and $f_{\mathrm{s}}(t_k) = 0$, then system (10) is represented as follows:

$$\begin{cases} \dot{e}(t) = (A-LC)e(t) - hLCu_1(t), \\ z(t) = \dot{e}(t), \end{cases} \tag{14}$$

which, in turn, can be rewritten as

$$z(t) = G(t)u_1(t), u_1(t) = \Delta z(t), \tag{15}$$

where the transfer function of $G(t)$ is $s(sI-(A-LC))^{-1}(-hLC)$. Let $R = I$ and $T \to \infty$ in Lemma 1, then $\|\Delta\|_\infty \leqslant 1$. Followed from the scaled small gain theorem[31], system (5) is asymptotically stable if there exists a nonsingular matrix $X \in \mathbb{R}^{n\times n}$ such that

$$\|G_X\|_\infty < 1, \tag{16}$$

where $G_X = XGX^{-1}$.

Consider the Lyapunov function $V(x) = e^{\mathrm{T}}(t)P_1 \cdot e(t)$ and introduce the following quadratic form:

$$W_d(e, u_1) = \dot{V}(e) + hz^{\mathrm{T}}Rz - hu_1^{\mathrm{T}}Ru_1. \quad (17)$$

It can be proved that for a some $\varepsilon > 0$ and any e and u_1, if

$$V(e) \geqslant \varepsilon \|e\|^2, \quad (18)$$

$$W_d(e, u_1) \leqslant -\varepsilon(\|e\|^2 + \|u_1\|^2), \quad (19)$$

then (16) holds. To show this, we integrate (19) from 0 to t and take into account $V(e(0)) = 0$ and then have

$$V(e(t)) + h\int_0^t [z^{\mathrm{T}}(\tau)Rz(\tau) - u_1^{\mathrm{T}}(\tau)Ru_1(\tau)]\mathrm{d}\tau \leqslant -\varepsilon\int_0^t (\|e\|^2 + \|u_1\|^2)\mathrm{d}\tau. \quad (20)$$

Let $\delta = \varepsilon/\lambda_{\max}(hR), X^{\mathrm{T}}X = hR$, it follows from (20) that

$$\int_0^t \|Xz(\tau)\|^2\mathrm{d}\tau - (1-\delta)\int_0^t \|Xu_1(\tau)\|^2\mathrm{d}\tau \leqslant \int_0^t [z^{\mathrm{T}}(\tau)(hR)z(\tau) - u_1^{\mathrm{T}}(\tau)(hR - \varepsilon I)u_1(\tau)]\mathrm{d}\tau \leqslant -\varepsilon\int_0^t \|e\|^2\mathrm{d}\tau - V(e(t)) \leqslant 0.$$

We thus have

$$\|G_X\|_\infty \leqslant (1-\delta) < 1. \quad (21)$$

Next we are seeking for conditions satisfying (18)(19). Obviously, if $P_1 > 0$, then (18) holds. Applying descriptor system approach[24], we can rewrite (17) as

$$\begin{aligned}
&W_d(e(t), u_1(t)) = \\
&\dot{V}(e(t)) + hz^{\mathrm{T}}(t)Rz(t) - hu_1^{\mathrm{T}}(t)Ru_1(t) = \\
&2e^{\mathrm{T}}(t)P_1\dot{e}(t) + hz^{\mathrm{T}}(t)Rz(t) - hu_1^{\mathrm{T}}(t)Ru_1(t) = \\
&hz^{\mathrm{T}}(t)Rz(t) - hu_1^{\mathrm{T}}(t)Ru_1(t) + \\
&2[e^{\mathrm{T}}(t)\ \ z^{\mathrm{T}}(t)]P^{\mathrm{T}}\begin{bmatrix} z(t) \\ 0 \end{bmatrix} = \\
&hz^{\mathrm{T}}(t)Rz(t) - hu_1^{\mathrm{T}}(t)Ru_1(t) + 2[e^{\mathrm{T}}(t)\ \ z^{\mathrm{T}}(t)]P^{\mathrm{T}} \cdot \\
&\begin{bmatrix} z(t) \\ -z(t) + (A - LC)e(t) - hLCu_1(t) \end{bmatrix} = \\
&[e^{\mathrm{T}}\ \ z^{\mathrm{T}}\ \ u_1^{\mathrm{T}}]\begin{bmatrix} \varPsi & hP^{\mathrm{T}}\begin{bmatrix} 0 \\ -LC \end{bmatrix} \\ * & -hR \end{bmatrix}\begin{bmatrix} e \\ z \\ u_1 \end{bmatrix},
\end{aligned}$$

where

$$\varPsi = P^{\mathrm{T}}\begin{bmatrix} 0 & I \\ A - LC & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A - LC & -I \end{bmatrix}^{\mathrm{T}} P + h\begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix},$$

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}.$$

Thus, a sufficient condition to satisfy (19) is

$$\begin{bmatrix} \varPsi & hP^{\mathrm{T}}\begin{bmatrix} 0 \\ -LC \end{bmatrix} \\ * & -hR \end{bmatrix} < 0. \quad (22)$$

So we can conclude that if (22) holds and $P_1 > 0$, then (5) is asymptotically stable.

Finally we prove the noise robustness constraint (6). Let $f_{\mathrm{s}} = 0$ and define the following quadratic form:

$$W_{\mathrm{r}} = W_d(e(t), u_1(t)) + r^{\mathrm{T}}(t)r(t) - \gamma^2[w^{\mathrm{T}}(t)w(t) + v^{\mathrm{T}}(t-\tau(t))(t-\tau(t))], \quad (23)$$

Integrating (23) from 0 to ∞ and following from Lemma 1, we see that if

$$W_{\mathrm{r}} < 0 \quad (24)$$

holds, then the noise robustness constraint (6) is satisfied. Note that

$$\begin{aligned}
&\dot{V} = 2e^{\mathrm{T}}(t)P_1\dot{e}(t) = \\
&2[e^{\mathrm{T}}(t)\ z^{\mathrm{T}}(t)]P^{\mathrm{T}} \cdot \\
&\begin{bmatrix} z(t) \\ -z(t) + (A - LC)e(t) + B_{\mathrm{w}}w(t) \\ -hLCu_1(t) + LDv(t - \tau(t)) \end{bmatrix},
\end{aligned} \quad (25)$$

Substituting $W_d(e(t), u_1(t))$ into (23) and applying Schur complements to the term $r^{\mathrm{T}}(t)r(t)$, we can obtain that (24) is satisfied if

$$\begin{bmatrix}
\varPsi & hP^{\mathrm{T}}\begin{bmatrix} 0 \\ -LC \end{bmatrix} & P^{\mathrm{T}}\begin{bmatrix} 0 \\ B_{\mathrm{w}} \end{bmatrix} & P^{\mathrm{T}}\begin{bmatrix} 0 \\ LD_{\mathrm{v}} \end{bmatrix} & \begin{bmatrix} C^{\mathrm{T}}H^{\mathrm{T}} \\ 0 \end{bmatrix} \\
* & -hR & 0 & 0 & hC^{\mathrm{T}}H^{\mathrm{T}} \\
* & * & -\gamma^2 I & 0 & 0 \\
* & * & * & -\gamma^2 I & D_{\mathrm{v}}^{\mathrm{T}}H^{\mathrm{T}} \\
* & * & * & * & -I
\end{bmatrix} < 0. \quad (26)$$

Note that (26) implies (23), namely (5) is asymptotically stable. MI (13) results from the latter LMI by expansion of the block matrices.

3.2 Fault sensitivity analysis

In this subsection, we design a fault detection filter under the noise free case ($w(t)=0, v(t_k)=0$) to satisfy the fault sensitivity condition (8). When there are no process and measurement noises, the error dynamics (5) can be expressed as

$$\begin{cases}\dot{e}(t)= \\ (A-LC)e(t)-hLCu_1(t)-LD_{\mathrm{f}}f_{\mathrm{s}}(t-\tau(t)), \\ r(t)= \\ HCe(t)+hHCu_1(t)+HD_{\mathrm{f}}f_{\mathrm{s}}(t-\tau(t)), \\ z(t)=\dot{e}(t),\end{cases} \tag{27}$$

where $u_1(t)$ is given by (11).

Theorem 2 Under the non-uniformly sampled-data system (1) and the fault detection filter (4) and assume $w(t)=0, v(t_k)=0$ (noise free) and let $W=H^{\mathrm{T}}H$. For a given scalar $\beta>0$, if there exist $Q_1>0$, Q_2, Q_3, $M>0$, L such that

$$\begin{bmatrix}\varTheta_1 & \varTheta_2 & \varTheta_3 & \varTheta_4 \\ * & \varTheta_5 & -hP_3^{\mathrm{T}}LC & -hQ_3^{\mathrm{T}}LD_{\mathrm{f}} \\ * & * & -hM-h^2C^{\mathrm{T}}WC & -hC^{\mathrm{T}}WD_{\mathrm{f}} \\ * & * & * & \beta^2I-D_{\mathrm{f}}^{\mathrm{T}}WD_{\mathrm{f}}\end{bmatrix}<0, \tag{28}$$

where

$$\varTheta_1=Q_2^{\mathrm{T}}(A-LC)+(A-LC)^{\mathrm{T}}Q_2-C^{\mathrm{T}}WC,$$
$$\varTheta_2=Q_1-Q_2^{\mathrm{T}}+(A-LC)^{\mathrm{T}}Q_3,$$
$$\varTheta_3=-hQ_2^{\mathrm{T}}LC-hC^{\mathrm{T}}WC,$$
$$\varTheta_4=-Q_2^{\mathrm{T}}LD_{\mathrm{f}}-C^{\mathrm{T}}WD_{\mathrm{f}},$$
$$\varTheta_5=-Q_3^{\mathrm{T}}-Q_3+hM.$$

Proof It can be derived along the same line as the proof of Theorem 1.

Remark 2 Seen from the term (4, 4) of (28), a necessary condition to satisfy the MI is $\beta^2I-D_{\mathrm{f}}^{\mathrm{T}}WD_{\mathrm{f}}<0$. Therefore, the fault sensitivity performance index β is less than $\|D_{\mathrm{f}}H\|$. As for the sensor fault distribution matrix $D_{\mathrm{f}}\neq 0$, which guarantees the proposed method can effectively detect the fault.

3.3 Filter gain iterative design

In previous two subsections, the sufficient conditions to satisfy the noise robustness constraint (6) and the fault sensitivity condition (8) have just been given in terms of MIs. An iterative LMI algorithm is proposed in this section to design the filter gain L which can make a tradeoff between noise robustness and fault sensitivity, namely make γ/β as small as possible.

Filter gain iterative design algorithm: Given system matrices $A, B_{\mathrm{u}}, B_{\mathrm{w}}, C, D_{\mathrm{v}}, D_{\mathrm{f}}$ and the post weighting matrix H and let $\mu_1>0, \mu_2>0$ be sufficiently small adjustable parameters. Set $i=0, j=0$ and $k\in\mathbb{Z}^+$ to be computational loops numbers.

Step 1 Choose a sufficiently large $\gamma=\varsigma$ and solve (13) to find L and let $\gamma=\varsigma$ and $\beta=0$.

Step 2 (Main Iterative Steps):

i) Substitute L into (13) and (28) and find a feasible solution set of P_1, P_2, P_3, R, Q_1, Q_2, Q_3, M.

ii) Set $i=i+1$, substitute P_1, P_2, P_3, R, Q_1, Q_2, Q_3, M obtained in last step into (13) and (28) and let $\gamma=\gamma-\mu_1$ and $\beta=\beta+\mu_2$, find a feasible solution L for LMIs (13) and (28). Store $L_i=L$ and γ/β, repeat Step2 i)ii) until there is no feasible solution. Then let $L_i=L_{i-1}$.

iii) If γ/β is less than the desired level, then a desired observer gain $L_i=L$ is found. Stop.

Step 3 Set $j=j+1$, if $j<k$, repeat Step 2, else stop (The feasible solution cannot be found).

Remark 3 In Step 1, MI (13) is still not LMI. Let $P_3=\varepsilon P_2$, where ε is a nonzero scalar. Note that $P_3^{\mathrm{T}}+P_3$ appears on the diagonal, the matrix P_2 is nonsingular. Define $P_2^{\mathrm{T}}L=\tilde{P}^{\mathrm{T}}$, MI (13) can be easily transformed into an LMI, and hence the filter gain matrix is $L=(P_2^{\mathrm{T}})^{-1}\tilde{P}^{\mathrm{T}}$.

Remark 4 In Step 2, for given a set of P_1, P_2, P_3, R, Q_1, Q_2, Q_3, M, MIs (13) and (28) becomes LMIs and a feasible solution L can always be obtained as long as μ_1 and μ_2 are sufficiently small. Therefore, Step 2 can always locally improve the performance of fault detection filter.

Remark 5 It is assumed that $w(t)$ belongs to $\mathrm{L}_2[0,\infty)$, and $v(t_k)$ belongs to $\mathrm{l}_2[0,\infty)$. Because residual evaluation over the whole time domain is unrealistic, we choose T as the length of the evaluation time window. If $\|w\|_{2,T}\leqslant w_0$, $\|v\|_{2,T}\leqslant v_0$ and the noise robustness constraint (6) is satisfied, then the residual $r(t)$ satisfies the following inequality

$$\|r(t)\|_{2,\,T}^2< \\ \gamma^2w_0^2+\gamma^2\sum_{k=0}^{T}(t_{k+1}-t_k)v^{\mathrm{T}}(t_k)v(t_k)\leqslant \\ \gamma^2w_0^2+\gamma^2hv_0^2,$$

for the fault free case. Therefore, the detection threshold can be chosen as

$$T_{\mathrm{r}} = \gamma w_0 + \gamma\sqrt{h}v_0. \tag{29}$$

The fault detection logic is

$$\|r(t)\| \geqslant T_{\mathrm{r}} \Rightarrow \text{Faulty} \Rightarrow \text{Alarm},$$
$$\|r(t)\| < T_{\mathrm{r}} \Rightarrow \text{No fault}.$$

4 Aircraft example

A remotely piloted aircraft example is examined in this section to illustrate the effectiveness of the proposed method. Consider the following four-state model of the linearized lateral dynamics corrupted by continuous-time process noise and discrete-time measurement noise[32]:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_{\mathrm{u}}u(t) + B_{\mathrm{w}}w(t), \\ y(t_k) = Cx(t_k) + D_{\mathrm{v}}v(t_k) + D_{\mathrm{f}}f_{\mathrm{s}}(t_k), \end{cases}$$

where $x = [\sigma \ \ p \ \ r \ \ \phi]^{\mathrm{T}}$ with σ being the sideslip, p the roll rate, r the yaw rate, ϕ the bank angle, $u(t) = [\delta_{\mathrm{r}} \ \ \delta_a]^{\mathrm{T}}$ is the control input with being δ_{r} the rudder and δ_a the aileron. The system measurements outputs $y = [p \ \ \phi]^{\mathrm{T}}$ are non-uniformly sampled by roll rate sensor and bank angle sensor. A fault occurs in the roll rate sensor. The system matrices are

$$A = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 \\ -0.1033 & -8.525 & 3.75 & 0 \\ 0.3649 & 0 & -0.639 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$B_{\mathrm{u}} = \begin{bmatrix} -5.432 & 0 \\ 0 & -28.64 \\ -9.49 & 0 \\ 0 & 0 \end{bmatrix}, B_{\mathrm{w}} = \begin{bmatrix} 0.1 \\ 1 \\ 0 \\ 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D_{\mathrm{f}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, D_{\mathrm{v}} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

Assume that the sample interval is a random variable obeying the uniform distribution on $(0.2, \ \ 0.5)$, and then we have $h = 0.5$. Let $H = [1 \ \ 0; 0 \ \ 1]$. Based on the iterative algorithm, we can obtain that γ/β approaches 0.87, where $\gamma = 0.67$ and $\beta = 0.77$, and the corresponding filter gain is

$$L = \begin{bmatrix} 0.0346 & 1.0071 \\ 0.0396 & 0.0650 \\ 0.0078 & 0.2083 \\ 0.0095 & 0.8085 \end{bmatrix}.$$

The control input is $u(t) = [1 \ \ 2]^{\mathrm{T}}$, w, v are zero mean white noises with variance 0.1 respectively, and the two kinds of faults (a constant fault and a time varying fault) are ,respectively, as follows

$$f_{s1}(t_k) = \begin{cases} 0, t_k < 10, \\ 1, 10 \leqslant t_k \leqslant 30, \end{cases}$$

$$f_{s2}(t_k) = \begin{cases} 0, & t_k < 10, \\ 1 + 0.5\sin(4\pi t_k), & 10 \leqslant t_k \leqslant 30. \end{cases}$$

Figures1 and 2 show the residuals under the cases of constant fault and time-varying fault over time window $[0, \ \ 40]$ s respectively. Choosing the evaluation time $T = 2$ s, it then follows from (29) that $T_{\mathrm{r}} = 0.114$. Fig.3 and 4 show the residual evaluations for the two cases. It can be seen that the designed fault detection filter can effectively detect the occurrence and disappearance of the fault despite the continuous time process noise and discrete time measurement noise.

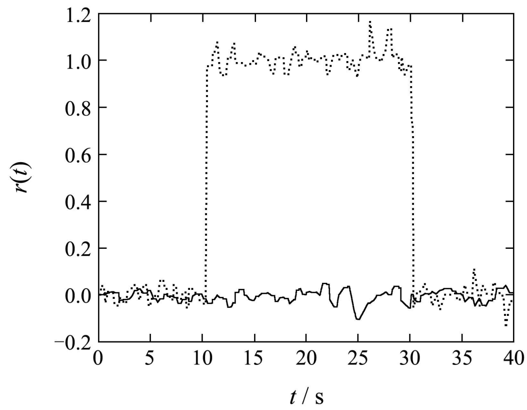

Fig. 1 The residual under constant fault case

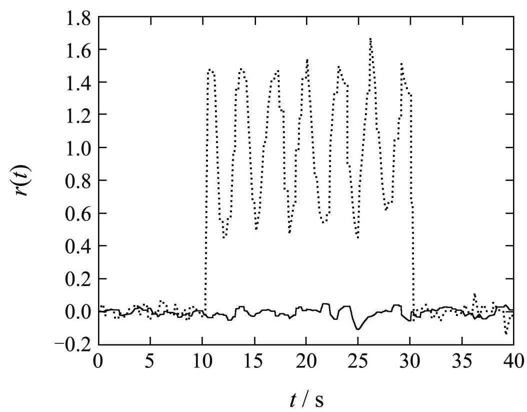

Fig. 2 The residual under time varying fault case

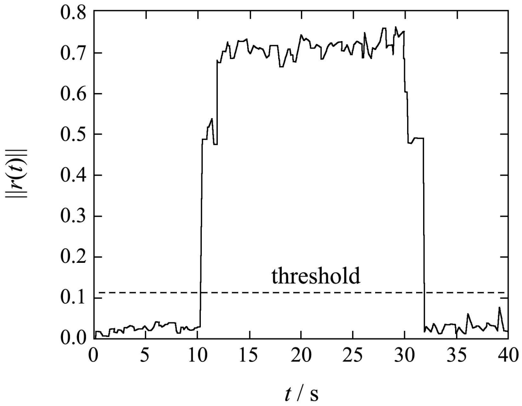

Fig. 3 The residual evaluation under constant fault case

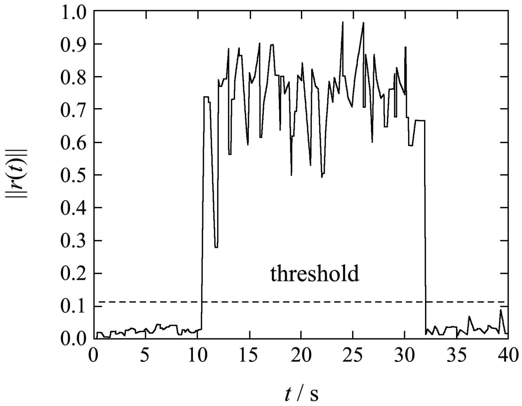

Fig. 4 The residual evaluation under time varying fault case

5 Conclusion

The problem of sensor fault detection of non-uniformly sampled-data systems has been addressed by using the output delay approach in this paper. Compared with the system considered in [21～23], the system in this paper is more general with no periodicity assumption and no strictly properness constraint. The designed fault detection filter not only guarantees the H_∞ norm from continuous time process noise and discrete time measurement noise to residual less than a prescribed value, but also ensures the l_2 gain from sensor fault to residual greater than a prescribed value. Meanwhile, the proposed iterative algorithm can make a good tradeoff between noise robustness and fault sensitivity. The effectiveness of the proposed design technique has been demonstrated on an aircraft example.

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