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Adaptive generalized predictive decoupling control for a class of MIMO nonlinear systems based on unmodeled dynamic compensation

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Abstract: For a class of uncertain multi-input-multi-output (MIMO) discrete-time nonlinear systems with strong coupling and unstable zero-dynamics, an adaptive generalized predictive decoupling switching control method based on unmodeled dynamic compensation is proposed. It is only required that the higher order nonlinear terms of the system to satisfy a linear growth condition, rather than the global boundedness condition widely used. The analysis of stability and convergence of the adaptive control method are performed. Moreover, in designing the nonlinear generalized predictive decoupling controller, we combine the adaptive-network-based fuzzy inference system (ANFIS) training with the "one-to-one mapping" technique to adaptively estimate the unmodeled dynamics, so that the universal approximation property of ANFIS can be guaranteed. Finally, simulation results demonstrate the superiority of the proposed method and validate the theoretical analysis.

Key words: generalized predictive control; decoupling; nonlinear systems; switching; unmodeled dynamic compensation

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基于未建模动态补偿的一类MIMO非线性系统的自适应广义预测解耦切换控制

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摘要: 针对一类不确定的多输入多输出(MIMO)离散时间零动态不稳定非线性系统, 提出了一种基于未建模动态补偿的非线性广义预测解耦切换控制方法. 该控制方法要求系统的未建模动态满足线性增长条件, 放宽了未建模动态全局有界的限制. 建立了所提的自适应控制方法的稳定性和收敛性分析. 而且, 在设计广义预测解耦控制器时, 把"一一映射"与ANFIS的训练相结合来估计系统的未建模动态, 保证了ANFIS的万能逼近特性. 最后, 仿真结果验证了所提方法的优越性.

关键词: 广义预测控制; 解耦; 非线性系统; 切换; 未建模动态补偿

1 Introduction

Most complex industrial processes have compositive complexities including multivariable, strong coupling, serious nonlinearity, large uncertainty and time delay. Especially, the strong couplings among different loops of such multivariable systems usually results in degradation of tracking performances, even leading to system instability. Adaptive decoupling control is one of the most effective ways to cope with this problem[1–4]. Indeed, decoupling control was initially developed for deterministic linear systems. To deal with coupling control in nonlinear systems, neural networks and fuzzy methods have been adopted[5–7]. As the generalized predictive control (GPC) using the tra-

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ditional parametric model, the parameters are comparatively few and easy to be estimated online to achieve the adaptive control. Moreover, GPC adopts multi-step prediction, rolling optimization and feedback correction strategy, compared with the conventional control methods, the abilities of GPC to resist load disturbance, random noise and time-delay variations are significantly enhanced. Hence, GPC has been paid much attention from both academic and industrial fields during the last few decades[8–11]. In [12], an adaptive generalized predictive decoupling controller based on neural networks is designed for nonlinear systems. However, it lacked the stability and convergence analysis of the closed-loop system. Multiple-model control approach which was developed by Narendra[13] provides an effective way to ensure the stability and desirable performance of the system. In [14], by using neural networks and multiple models, the authors proposed an adaptive control framework. Later on, many efforts have been made under this framework[15–17]. Recently, the framework is extended from the originally SISO discrete-time nonlinear systems to MIMO case[18–19]. However, in [14–19], the unmodeled dynamics of the system was assumed to be globally bounded, which has inevitably led to considerable conservatism for multiple-model theory and its applications. Indeed, once it has been assumed that the unmodeled dynamics are bounded, it can be treated as a bounded disturbance to the system, where standard analysis and design tools can be applied to obtain the required controller. Such an assumption is not valid in general because the higher order terms should also depend on the variables (inputs and outputs) of the system which are not bounded before the controller design. Therefore, it is necessary to remove such a strict assumption and simply assume that the unmodeled dynamics are linearly unbounded. This is one of the key issues that will be addressed in this paper, where an ANFIS[20] is adopted to estimate the unmodeled dynamics. This is because the parameters in ANFIS have specific physical meanings and are obtained by the hybrid algorithm which consists of the gradient descent and the least-squares estimation. It has been shown that such an algorithm can improve the convergence rate and reduce the possibility for the network being trapped in local minima[20]. Moreover, to fully use the universal approximation property of neural networks, a one-to-one mapping was proposed in [21], where the idea was to use a simple one-to-one mapping to project the input variables of the neural network model into a bounded set and then use neural network to approximate the concerned nonlinear function. Such a technique will also be used in this paper, so that the universal approximation property of ANFIS can be guaranteed.

Given the above analysis, this paper requires the higher order nonlinear terms of the systems to satisfy a linear growth condition, an adaptive generalized predictive decoupling control method based on unmodeled dynamic compensation is proposed. The analysis of stability and convergence of the adaptive decoupling control method are established.

2 Problem statement

Consider a class of multivariables discrete-time nonlinear systems described as:

$$A(z^{-1})y(k) = z^{-1}\bar{B}(z^{-1})u(k)+z^{-1}\bar{\bar{B}}(z^{-1})u(k) + v[X(k-1)] + d(k), \quad (1)$$

where $u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^m$ are the system input and output at time k, respectively; $d(k)$ denotes a bounded disturbance, which includes the measurement noise and unmeasurement disturbance, and is assumed to be bounded and uncorrelated with the inputs[22] i.e., $\|d(k)\| \leqslant d_0$, where $d_0 > 0$ is a known constant. $A(z^{-1}), \bar{B}(z^{-1})$ and $\bar{\bar{B}}(z^{-1})$ are polynomial matrixes in terms of the unit back shift operator z^{-1} with $A(z^{-1})$ and $\bar{B}(z^{-1})$ are being diagonal, and $\bar{\bar{B}}(z^{-1})$ is a polynomial matrix with zero diagonal elements. Define

$$\bar{B}(z^{-1}) = \mathrm{diag}\{B_{ii}(z^{-1})\},\ i = 1, \cdots, m,$$
$$\bar{\bar{B}}(z^{-1}) = B(z^{-1}) - \bar{B}(z^{-1}).$$

The definitions of $A(z^{-1}), B(z^{-1})$ and $v[\,\cdot\,]$ are given by

$$A(z^{-1}) = I - A_1 z^{-1} - \cdots - A_n z^{-n},$$

where I is an identity matrix.

$$B(z^{-1}) = \bar{B}(z^{-1}) + \bar{\bar{B}}(z^{-1}) = B_0 + B_1 z^{-1} + \cdots + B_{n-1} z^{-n+1},$$

$v[X(k)]$ is a continuous vector-valued nonlinear function, which can be described by

$$v[\,\cdot\,] := v[X(k)] = A(z^{-1})y(k+1) - B(z^{-1})u(k) - d(k),$$

where $X(k) := [y(k)^{\mathrm{T}} \ \cdots \ y(k+1-n)^{\mathrm{T}} \ u(k)^{\mathrm{T}} \ \cdots \ u(k+1-n)^{\mathrm{T}}]^{\mathrm{T}}$.

The objective of this paper is to design an adaptive generalized predictive decoupling controller such that the following requirements are satisfied:

1) all the signals in the closed-loop system are uniformly bounded;

2) the output $y(k)$ asymptotically tracks a specified bounded signal $w(k)$ so that the influence of the coupling and nonlinearity can be minimized.

Condition 1 It is assumed that the nonlinear term $v[X(k)]$ satisfies the following condition:

$$\|v[X(k)]\| \leqslant \gamma(k),\ \forall k,$$

where $\gamma(k) = \varepsilon_1\|X(k)\| + \varepsilon_2$, therein $0 < \varepsilon_1 < 1$, $\varepsilon_2 > 0$ are known constants.

3 Nonlinear generalized predictive decoupling controller based on compensation of the unmodeled dynamics

For the nonlinear controlled plant (1), a nonlinear generalized predictive decoupling controller which contains a feedback controller, a decoupling compensator and feedforward compensator shown in Fig.1 is proposed. Here the feedback controller consists of the polynomial matrixes $P(z^{-1})$, $H_{\rm c}(z^{-1})$ and $F_{\rm c}(z^{-1})$ of z^{-1}. It is expected that such a feedback controller can be used to realize asymptotically tracking of the output $y(k)$ with respect to a specified bounded signal $w(k)$. A nonlinear decoupling compensator denoted by $\bar{\bar{M}}_{\rm c}(z^{-1})$ is a polynomial matrix with zero diagonal elements. It can be used to eliminate the influence of the coupling term of the system. A nonlinear compensator $M_{\rm c}(z^{-1})$ is a polynomial matrix of z^{-1} and is used to eliminate the influence from the nonlinear term $v[\,\cdot\,]$ to the controlled output.

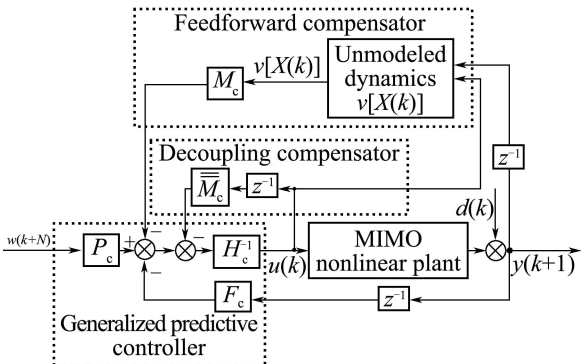

Fig. 1 Nonlinear generalized predictive decoupling controller based on compensation of unmodeled dynamics

From Fig.1, it can be seen that the control input $u(k)$ can be expressed as

$$u(k) = H_{\rm c}^{-1}(z^{-1})\{P_{\rm c}(z^{-1})w(k+N) - F_{\rm c}(z^{-1})y(k) - \bar{\bar{M}}_{\rm c}(z^{-1})u(k-1) - M_{\rm c}(z^{-1})v[X(k)]\}. \tag{2}$$

By substituting Eq.(2) into Eq.(1), we obtain

$$[H_{\rm c}(z^{-1})A(z^{-1}) + z^{-1}\bar{B}(z^{-1})F_{\rm c}(z^{-1})]y(k) = z^{-1}\bar{B}(z^{-1})P_{\rm c}(z^{-1})w(k+N) + [H_{\rm c}(z^{-1})\bar{\bar{B}}(z^{-1}) - z^{-1}\bar{B}(z^{-1})\bar{\bar{M}}_{\rm c}(z^{-1})] \times u(k-1) + [H_{\rm c}(z^{-1}) - \bar{B}(z^{-1})M_{\rm c}(z^{-1})] \times v[X(k-1)] + H_{\rm c}(z^{-1})d(k). \tag{3}$$

From Eq.(3), it can be seen that $[H_{\rm c}(z^{-1})A(z^{-1}) + z^{-1}\bar{B}(z^{-1})F_{\rm c}(z^{-1})]$ and $\bar{B}(z^{-1})P_{\rm c}(z^{-1})$ are all diagonal polynomial matrices. $[H_{\rm c}(z^{-1})\bar{\bar{B}}(z^{-1}) - z^{-1} \times \bar{B}(z^{-1})\bar{\bar{M}}_{\rm c}(z^{-1})]u(k-1)$ and $[H_{\rm c}(z^{-1}) - \bar{B}(z^{-1}) \times M_{\rm c}(z^{-1})]v[X(k-1)]$ are the coupling term of the system. By appropriately choosing the polynomials matrixes $P(z^{-1}), H_{\rm c}(z^{-1}), F_{\rm c}(z^{-1})$ and $M_{\rm c}(z^{-1})$, one can realize the output $y(k)$ tracking the reference input $w(k)$ and minimize the influence of coupling term and the unmodeled dynamics $v[\,\cdot\,]$ on the output of the system.

In order to choose the parameters matrixes $P(z^{-1})$, $H_{\rm c}(z^{-1}), F_{\rm c}(z^{-1})$ and $M_{\rm c}(z^{-1})$ of the controller, the following performance index is introduced[12]:

$$J = \sum_{j=1}^{N} \|y(k+j) - R_j w(k+j) + K_j(z^{-1})u(k+j-1) + S_j(z^{-1})v[X(k+j-1)]\|_{q_j}^2 + \sum_{j=1}^{N_u} \|u(k+j-1)\|_{\lambda_j}^2, \tag{4}$$

where $w(k) \in \mathbb{R}^n$ is a known bounded reference input signal which stands for the ideal output of the system. N and N_u are the predictive horizon and the control horizon, respectively; R_j, q_j and λ_j are diagonal weighting matrices; $K_j(z^{-1})$ is a weighting polynomial matrix with zero diagonal elements; $S_j(z^{-1})$ is a diagonal weighting polynomial matrix of z^{-1}.

In order to obtain the jth step-ahead prediction of $y(k)$, the following two Diophantine equations are introduced:

$$I = E_j(z^{-1})A(z^{-1}) + z^{-j}F_j(z^{-1}), \tag{5}$$

$$E_j(z^{-1})\bar{B}(z^{-1}) = G_j(z^{-1}) + z^{-j}H_j(z^{-1}), \tag{6}$$

$$E_j(z^{-1})\bar{\bar{B}}(z^{-1}) = \bar{\bar{G}}_j(z^{-1}) + z^{-j}\bar{\bar{H}}_j(z^{-1}), \tag{7}$$

where $E_j(z^{-1})$, $F_j(z^{-1})$, $G_j(z^{-1})$ and $H_j(z^{-1})$ are all diagonal weighting polynomial matrix of z^{-1}; $\bar{\bar{G}}_j(z^{-1})$ and $\bar{\bar{H}}_j(z^{-1})$ are polynomial matrix with zero diagonal elements.

From Eqs. (1), (5)−(7), the following jth step-ahead output optimal prediction can be obtained as follows:

$$y(k+j)=$$
$$F_j(z^{-1})y(k)+G_j(z^{-1})u(k+j-1)+$$
$$H_j(z^{-1})u(k-1)+\bar{\bar{G}}_j(z^{-1})u(k+j-1)+$$
$$\bar{\bar{H}}_j(z^{-1})u(k-1)+E_j(z^{-1})v[X(k+j-1)]. \quad (8)$$

Substituting (8) into (4) and choosing the weighting polynomials $K_j(z^{-1}),\ j=1,\cdots,N$[23] yieldes

$$[K_j(z^{-1})+\bar{\bar{G}}_j(z^{-1})]u(k+j-1)+$$
$$\bar{\bar{H}}_j(z^{-1})u(k-1)=\bar{\bar{M}}_j(z^{-1})u(k-1),$$

where $\bar{\bar{M}}_j(z^{-1})=\bar{\bar{M}}_{j,0}+\bar{\bar{M}}_{j,1}z^{-1}+\cdots+\bar{\bar{M}}_{j,n}z^{-n_m}$ is polynomial matrix with zero diagonal elements. Choosing the weighting polynomials $S_j(z^{-1}), j=1,\cdots,N$[23] makes

$$[E_j(z^{-1})+S_j(z^{-1})]v[X(k+j-1)]=$$
$$M_j(z^{-1})v[X(k)],$$

where $M_j(z^{-1}) = M_{j,0} + M_{j,1}z^{-1} + \cdots + M_{j,n_m}z^{-n_m}$. Then, it can be obtained that

$$J=\sum_{j=1}^{N}\|F_j(z^{-1})y(k)+G_j(z^{-1})u(k+j-1)+$$
$$[H_j(z^{-1})+\bar{\bar{M}}_j(z^{-1})]u(k-1)+$$
$$M_j(z^{-1})v[X(k)]-R_jw(k+j)\|^2_{q_j}+$$
$$\sum_{j=1}^{N_u}\|u(k+j-1)\|^2_{\lambda_j}. \quad (9)$$

Eq. (9) can be rewritten in the following vector form:

$$J=\|Fy(k)+GU+(H+\bar{\bar{M}})u(k-1)+$$
$$Mv[X(k)]-RW\|^2_Q+\|U\|^2_\lambda, \quad (10)$$

where $U=[u(k)\ \cdots\ u(k+N_u-1)]^{\mathrm T}$, $W=[w(k+1)\ \cdots\ u(k+N)]^{\mathrm T}$, $H=[H_1(z^{-1})\ \cdots\ H_N(z^{-1})]^{\mathrm T}$, $F=[F_1(z^{-1})\ \cdots\ F_N(z^{-1})]^{\mathrm T}, R=\mathrm{diag}\{r_j\}$, $Q=\mathrm{diag}\{q_j\}$, $M=[M_1(z^{-1})\ \cdots\ M_N(z^{-1})]^{\mathrm T}$, $\bar{\bar{M}}=[\bar{\bar{M}}_1(z^{-1})\ \cdots\ \bar{\bar{M}}_N(z^{-1})]^{\mathrm T}, \lambda=\mathrm{diag}\{\lambda_1,\cdots,\lambda_N\}$, $j=1,\cdots,N$, G is a lower triangular Toeplitz matrix which is composed of the coefficients of $G_j(z^{-1})$.

It can be easily proved that the optimal generalized predictive decoupling controller that minimizes (10) is described by

$$U=(G^{\mathrm T}QG+\lambda)^{-1}G^{\mathrm T}Q[RW-Fy(k)-$$
$$Hu(k-1)-\bar{\bar{M}}u(k-1)-Mv[X(k)]]. \quad (11)$$

Defining the 1st n rows of matrix $(G^{\mathrm T}QG+\lambda)^{-1}G^{\mathrm T}Q$ as $P^{\mathrm T}=[p_1\ \cdots\ p_N]$, we formulate the nonlinear generalized predictive decoupling controller as

$$H_{\mathrm c}(z^{-1})u(k)=$$
$$P_{\mathrm c}(z^{-1})w(k+N)-F_{\mathrm c}(z^{-1})y(k)-$$
$$\bar{\bar{M}}_{\mathrm c}(z^{-1})u(k-1)-M_{\mathrm c}(z^{-1})v[X(k)], \quad (12)$$

where

$$P_{\mathrm c}(z^{-1})=\sum_{k=0}^{N-1}p_{N-k}R_{N-k}z^{-k},$$
$$F_{\mathrm c}(z^{-1})=\sum_{k=1}^{N}p_kF_k(z^{-1}),$$
$$H_{\mathrm c}(z^{-1})=I+z^{-1}\sum_{k=1}^{N}p_kH_k(z^{-1}),$$
$$\bar{\bar{M}}_{\mathrm c}(z^{-1})=\sum_{k=1}^{N}p_k\bar{\bar{M}}_k(z^{-1}),$$
$$M_{\mathrm c}(z^{-1})=\sum_{k=1}^{N}p_kM_k(z^{-1}).$$

When $v[\,\cdot\,]$ is small and can be neglected, one can then simply design a linear generalized predictive decoupling controller for the system (1) as follows:

$$H_{\mathrm c}(z^{-1})u(k)=P_{\mathrm c}(z^{-1})w(k+N)-F_{\mathrm c}(z^{-1})y(k)-$$
$$\bar{\bar{M}}_{\mathrm c}(z^{-1})u(k-1). \quad (13)$$

To facilitate the design of the controller, substituting (12) into (1) and eliminate $y(k)$, we can obtain the input of the system as

$$\{A(z^{-1})[H_{\mathrm c}(z^{-1})+z^{-1}\bar{\bar{M}}_{\mathrm c}(z^{-1})]+$$
$$z^{-1}F_{\mathrm c}(z^{-1})B(z^{-1})\}u(k)=$$
$$A(z^{-1})P_{\mathrm c}(z^{-1})w(k+N)-[z^{-1}F_{\mathrm c}(z^{-1})+$$
$$A(z^{-1})M_{\mathrm c}(z^{-1})]v[X(k)]-F_{\mathrm c}(z^{-1})d(k). \quad (14)$$

From (14), it can be seen that the polynomials matrices of the system input is

$$\{A(z^{-1})[H_{\mathrm c}(z^{-1})+z^{-1}\bar{\bar{M}}_{\mathrm c}(z^{-1})]+$$
$$z^{-1}F_{\mathrm c}(z^{-1})B(z^{-1})\},$$

which is related to the $H_{\mathrm c}(z^{-1})$, $\bar{\bar{M}}_{\mathrm c}(z^{-1})$ and $F_{\mathrm c}(z^{-1})$, depending on Q and λ. Therefore, in order to ensure the system stability, q_j and λ_j should be firstly chosen by a trial and error method to satisfy the following condition:

$$\det\{T(z^{-1})\}=$$
$$\det\{z^{-1}F_{\mathrm c}(z^{-1})B(z^{-1})+$$
$$A(z^{-1})[H_{\mathrm c}(z^{-1})+z^{-1}\bar{\bar{M}}_{\mathrm c}(z^{-1})]\}\neq 0,\ |z|\geqslant 1.$$
$$(15)$$

From the output equation (3), in order to eliminate the impact of coupling term, the following conditions should be satisfied.

$$H_{\mathrm c}(z^{-1})\bar{\bar{B}}(z^{-1})=z^{-1}\bar{B}(z^{-1})\bar{\bar{M}}_{\mathrm c}(z^{-1}), \quad (16)$$
$$H_{\mathrm c}(z^{-1})=\bar{B}(z^{-1})M_{\mathrm c}(z^{-1}). \quad (17)$$

Moreover, when $k\rightarrow\infty$, Eqs.(16)−(17) become

$$H_{\rm c}(1)\bar{\bar{B}}(1)=\bar{B}(1)\bar{\bar{M}}_{\rm c}(1), \quad (18)$$

$$H_{\rm c}(1)=\bar{B}(1)M_{\rm c}(1). \quad (19)$$

Then, the steady-state decoupling can be achieved if the following equation holds:

$$H_{\rm c}(1)A(1)+F_{\rm c}(1)\bar{B}(1)=\bar{B}(1)P(1). \quad (20)$$

The steay-state tracking error can be eliminated in the ideal case $d(k)=0$.

Remark 1 It should be noted that for a class of uncertain multivariable discrete-time nonlinear systems with unstable zero-dynamics, q_j and λ_j are not certainly existing. Therefore, the algorithm which proposed in this paper can only deal with a class of systems of which the parameter q_j and λ_j are existing.

4 Nonlinear adaptive generalized predictive decoupling control

4.1 Algorithm of adaptive generalized predictive decoupling control

Eq.(1) can be viewed as the estimation equation of the system parameters, it can be equivalently represented as follows:

$$y(k)-\hat{v}[X(k-1)]=\Theta^{\rm T}X(k-1)+\pi[X(k-1)], \quad (21)$$

where $\Theta=[A_1\ \cdots\ A_n\ B_0\ \cdots\ B_{n-1}]^{\rm T}$, $\pi[\,\cdot\,]=v[\,\cdot\,]-\hat{v}[\,\cdot\,]$. Please note that $\hat{v}[\,\cdot\,]$ is the estimation of $v[\,\cdot\,]$ by an ANFIS.

For Eq. (21), two estimation models are used. The first one is a linear estimation model given by

$$\hat{y}_1(k)=\hat{\Theta}_1^{\rm T}(k-1)X(k-1), \quad (22)$$

where $\hat{\Theta}_1^{\rm T}(k-1)$ is the estimation of the parameter vector Θ based on the linear model at instant $k-1$ and is updated by the following algorithm[14]

$$\hat{\Theta}_1(k)=\text{proj}\{\hat{\Theta}_1'(k)\}=\begin{cases}\hat{\Theta}_1'(k),\ \hat{H}_{1,0}(0)\neq0,\\ [\hat{A}_{1,0}(k)\ \cdots\ \hat{B}_{1,0}(k-1)\ \cdots\ \hat{B}_{1,n-1}(k)]^{\rm T}, \\ \qquad\text{otherwise},\end{cases} \quad (23)$$

$$\hat{\Theta}_1'(k)=\hat{\Theta}_1(k-1)+\frac{\mu_1(k)X(k-1)e_1^{\rm T}(k)}{1+X(k-1)^{\rm T}X(k-1)}, \quad (24)$$

$$\mu_1(k)=\begin{cases}1, & \|e_1(k)\|>4\gamma(k-1),\\ 0, & \text{otherwise},\end{cases} \quad (25)$$

where $\gamma(k-1)$ is an upper boundary function of the nonliear term $v[X(k)]$. $e_1(k)$ is the estimation error which is calculated from

$$\begin{aligned}&e_1(k)=y(k)-\hat{y}_1(k)=\\&y(k)-\hat{\Theta}_1^{\rm T}(k-1)X(k-1), \quad (26)\\&\hat{\Theta}_1'(k)=[\hat{A}_{1,0}(k)\ \cdots\ \hat{A}_{1,n-1}(k)\\&\qquad \hat{B}_{1,0}'(k)\ \cdots\ \hat{B}_{1,n-1}(k)]^{\rm T}.\end{aligned}$$

Noting

$$\begin{aligned}&\hat{A}_1(k,z^{-1})=\\&I-\hat{A}_{1,0}(k)z^{-1}-\cdots-\hat{A}_{1,n-1}(k)z^{-n+1},\\&\hat{B}_1(k,z^{-1})=\\&\hat{B}_{1,0}(k)+\hat{B}_{1,0}(k)z^{-1}+\cdots+\hat{B}_{1,n-1}(k)z^{-n+1}.\end{aligned}$$

According to (13) and the certainty equivalent principle, the linear adaptive generalized predictive decoupling controller $u_1(k)$ can be computed from

$$\begin{aligned}&\hat{H}_{1,\rm c}(z^{-1})u(k)=\\&\hat{P}_{1,\rm c}(z^{-1})w(k+N)-\hat{F}_{1,\rm c}(z^{-1})y(k)-\\&\hat{\bar{\bar{M}}}_{1,\rm c}(z^{-1})u(k-1), \quad (27)\end{aligned}$$

where $\hat{H}_{1,\rm c}(z^{-1}),\hat{P}_{1,\rm c}(z^{-1}),\hat{F}_{1,\rm c}(z^{-1})$ and $\hat{\bar{\bar{M}}}_{\rm c}(z^{-1})$ are obtained online by the identification Algorithm 1 and Eqs.(5)−(7).

When the system operates in a large range, a nonlinear estimation model should be used based on ANFIS. This estimation model is defined as follows:

$$\hat{y}_2(k)-\hat{v}[\,\cdot\,]=\hat{\Theta}_2^{\rm T}(k-1)X(k-1)+\pi[\,\cdot\,], \quad (28)$$

where $\hat{\Theta}_2(k)=[\hat{A}_{2,0}(k)\ \cdots\ \hat{A}_{2,n-1}(k)\ \hat{B}_{2,0}(k)\ \cdots\ \hat{B}_{2,n-1}(k)]^{\rm T}$ and its identification algorithm are similar to Algorithm 1 which are given as follows:

$$\hat{\Theta}_2(k)=\text{proj}\{\hat{\Theta}_2'(k)\}=\begin{cases}\hat{\Theta}_2'(k),\ \{\hat{H}_{1,0}(0)\}\neq0,\\ [\hat{A}_{2,0}(k)\ \cdots\ \hat{B}_{2,0}(k-1)\ \cdots\ \hat{B}_{2,n-1}(k)]^{\rm T}, \\ \qquad\text{otherwise}.\end{cases} \quad (29)$$

$$\hat{\Theta}_2'(k)=\hat{\Theta}_2(k-1)+\frac{a(k)X(k-1)e_2^{\rm T}(k)}{1+X(k-1)^{\rm T}X(k-1)},$$

$$a(k)=\begin{cases}1, & \|e_2(k)\|>4\xi;\\ 0, & \text{otherwise}.\end{cases} \quad (30)$$

$$\begin{aligned}&e_2(k)=y(k)-\hat{y}_2(k)=\\&y(k)-\hat{\Theta}_2^{\rm T}(k-1)X(k-1)-\hat{v}[X(k-1)]. \quad (31)\end{aligned}$$

Here ξ is a known positive constant which satisfies the condition $\|v[\,\cdot\,]-\hat{v}[\,\cdot\,]\|<\xi$; $\hat{v}[\,\cdot\,]$ is the estimation for $v[\,\cdot\,]$ by ANFIS at instant k. Taking into account that $v_i[\,\cdot\,]$ might be unbounded so that ANFIS cannot approach it, the following technique has to be used to solve this problem. Let

$$\varsigma[X(k)] = \frac{v_i[X(k)]}{\varepsilon_1\|X(k)\| + \varepsilon_2},$$

then it can be seen that $\|\varsigma[X(k)]\| \leqslant 1$. Since the boundedness of the definition domain $X(k)$ is unknown, even though ς is bounded in the limit, no guarantee can be made on the universal approximation ability of neural networks; that is, this function may not be able to satisfactorily approximated by neural networks in the whole input space, because of the $u(k)$ and $y(k)$ defined on $\Omega_i = (-\infty, +\infty)$. By using the one-to-one mapping α in [21]

$$\tilde{X}_i(k) = \alpha[X_i(k)] = \frac{1}{1+\exp[-X_i(k)]},\ i = 2n,$$

where α maps the interval $\Omega_i = (-\infty, +\infty)$ to $\tilde{\Omega}_i = (0,1)$ which can be further covered by the interval $\bar{\Omega}_i = [0,1]$. That is to say, the new variable $\tilde{X}(k) = [\tilde{X}_1(k)\ \tilde{X}_2(k)\ \cdots\ \tilde{X}_{2n}(k)] = [\tilde{y}(k)\ \cdots\ \tilde{y}(k-n+1)\ \tilde{u}(k)\ \cdots\ \tilde{u}(k-n+1)]$ is ranged in a closed bounded set. Therefore, the ANFIS can be firstly used to approximate the new function $\|\varsigma[\tilde{X}(k)]\|$. This means that $\hat{v}[X(k)]$ can be calculated by using

$$\hat{v}[X(k)] = \hat{\varsigma}[\tilde{X}(k)][\varepsilon_1\|\alpha^{-1}[\tilde{X}(k)]\| + \varepsilon_2], \quad (32)$$

where $\alpha^{-1}[\tilde{X}(k)] = [\alpha^{-1}(\tilde{X}_1(k))\ \cdots\ \alpha^{-1}(\tilde{X}_{2n}(k))]$.

At every instant k, using the similar formulation to those in [24], $v[\cdot]$ can be approximated online by an ANFIS. In order to prevent duplicates and save space, the details is omitted here, interested readers may refer to [24].

Noting

$$\hat{A}_2(k, z^{-1}) = 1 - \hat{A}_{2,0}(k)z^{-1} - \cdots - \hat{A}_{2,n-1}(k)z^{-n+1},$$
$$\hat{B}_2(k, z^{-1}) = \hat{B}_{2,0}(k) + \hat{B}_{2,1}(k)z^{-1} + \cdots + \hat{B}_{2,n-1}(k)z^{-n+1}.$$

According to Eqs.(12) and (32) and using the certainty equivalence principle, the nonlinear generalized predictive decoupling controller based on ANFIS is determined as

$$\hat{H}_{2,c}(z^{-1})u(k) = \hat{P}_{2,c}(z^{-1})w(k+N) - \hat{F}_{2,c}(z^{-1})y(k) - \hat{\bar{\bar{M}}}_{2,c}(z^{-1})u(k-1) - \hat{M}_{2,c}(z^{-1})\hat{v}[X(k)], \quad (33)$$

where $\hat{H}_{2,c}(z^{-1}),\ \hat{P}_{2,c}(z^{-1}),\ \hat{F}_{2,\,c}(z^{-1}),\ \hat{\bar{\bar{M}}}_{2,c}(z^{-1})$ and $\hat{M}_{2,c}(z^{-1})$ are obtained online by the identification Algorithm 2 and Eqs.(5)−(7).

To ensure the stability of the closed-loop at every instant k, the $\hat{Q}$ and $\hat{\lambda}$ should be calculated by the following equation:

$$T(z^{-1}) = z^{-1}\hat{F}_{ic}(z^{-1})\hat{B}_i(z^{-1}) + \hat{A}_i(z^{-1})[\hat{H}_{ic}(z^{-1}) + z^{-1}\hat{\bar{\bar{M}}}_{i,c}(z^{-1})],$$

where $i = 1, 2, T(z^{-1})$ is a specified stable polynomial matrix.

By Eqs.(18)−(20), in order to achieve steady decoupling and eliminate the steady state error, the weighting matrixes $K_j(z^{-1}),\ S_j(z^{-1})$, and R_j are chosen online to satisfy the following conditions:

$$\hat{A}_i(1)\hat{H}_{i,c}(1) + \hat{\bar{B}}_i(1)\hat{\bar{F}}_{i,c}(1) = \hat{\bar{B}}_i(1)\hat{P}_{i,c}(1), \quad (34)$$

$$\hat{H}_{i,c}(1)\hat{\bar{\bar{B}}}_i(1) = \hat{\bar{B}}_i(1)\hat{\bar{\bar{M}}}_{i,c}(1), \quad (35)$$

$$\hat{H}_{i,c}(1) = \hat{\bar{B}}_i(1)\hat{M}_{i,c}(1)\ ,\ i = 1, 2. \quad (36)$$

4.2 Switching mechanism

Similar to [14], a switching function is obtained as follows:

$$J_j(k) = \sum_{l=1}^{k} \frac{\mu_j(l)[\|e_j(l)\|^2 - 16\gamma(l-1)^2]}{4[1 + X(l-1)^{\mathrm{T}}X(l-1)]} + c\sum_{l=k-N+1}^{k}[1-\mu_j(l)]\|e_j(l)\|^2, \quad (37)$$

$$\mu_j(k) = \begin{cases} 1, & \|e_j(k)\| > 4\gamma(k-1),\ j = 1, 2, \\ 0, & \text{otherwise}, \end{cases} \quad (38)$$

where $j = 1$ denotes the linear decoupling controller, but $j = 2$ stands for the nonlinear decoupling controller. N is a positive integer. $c \geqslant 0$ is a predefined constant. $e_1(k)$ and $e_2(k)$ are obtained by (26) and (31), respectively.

At every instant k, the switching mechanism chooses the control law $u^*(k)$ corresponding to the smaller $J^*(k)$ to be applied to the system, i.e.,

$$J^*(k) = \min[J_1(k), J_2(k)].$$

If $J^*(k) = J_1(k),\ u_1(k)$ will be chosen to control the system;

If $J^*(k) = J_2(k),\ u_2(k)$ will be chosen to control the system.

5 Stability and convergence analysis

In order to prove the stability and the convergence of the proposed method in this paper, we need the following lemmas.

Lemma 1 Estimation algorithms (22)−(27) and (28)−(33) have the following properties:

ia) $\lim\limits_{k\to\infty}\dfrac{\mu_1(k)[\|e_1(k)\|^2-16\gamma(k-1)^2]}{4(1+X(k-1)^{\mathrm{T}}X(k-1))}=0$,

ib) $\lim\limits_{k\to\infty}\dfrac{a(k)[\|e_1(k)\|^2-16\xi^2]}{4(1+X(k-1)^{\mathrm{T}}X(k-1))}=0$,

ii) $\|\hat{\Theta}_i(k)-\Theta\|\leqslant\|\hat{\Theta}_i(0)-\Theta\|$,

iii) $\lim\limits_{k\to\infty}\|\hat{\Theta}_i(k)-\hat{\Theta}_i(k-\bar{k})\|=0$,

where $\bar{k}$ is an finite positive number and $i=1,2$ denotes Algorithm 1 and Algorithm 2, respectively.

Proof Define $\tilde{\Theta}(k)=\hat{\Theta}(k)-\Theta$. Similar to [14], the proof is straightforward and is omitted here due to space limitations.

Lemma 2 When the algorithm of linear adaptive generalized predictive decoupling control (22)−(27) and nonlinear adaptive generalized predictive decoupling control (28)−(33) are applied to system (1), respectively, the input-output dynamics of the system are given by

$$\begin{aligned}&\begin{bmatrix}T(z^{-1})+\Gamma_{11} & \Gamma_{12}\\ \Gamma_{14} & \hat{T}(z^{-1})+\Gamma_{15}\end{bmatrix}\begin{bmatrix}u(k)\\ y(k)\end{bmatrix}=\\&\begin{bmatrix}\hat{A}_1(z^{-1})\hat{P}_{1c}(z^{-1})+\Gamma_{13}\\ \tilde{B}_1(z^{-1})\hat{P}_{1c}(z^{-1})+\Gamma_{16}\end{bmatrix}w(k+N)+\\&\begin{bmatrix}-\hat{F}_{1,c}(z^{-1})\\ \tilde{Q}_1(z^{-1})\end{bmatrix}e_1(k),\end{aligned}\tag{39}$$

$$\begin{aligned}&\begin{bmatrix}T(z^{-1})+\Gamma_{21} & \Gamma_{22}\\ \Gamma_{25} & \hat{T}(z^{-1})+\Gamma_{26}\end{bmatrix}\begin{bmatrix}u(k)\\ y(k)\end{bmatrix}=\\&\begin{bmatrix}\hat{A}_2(z^{-1})\hat{P}_{2c}(z^{-1})+\Gamma_{23}\\ \tilde{B}_2(z^{-1})\hat{P}_{2c}(z^{-1})+\Gamma_{27}\end{bmatrix}w(k+N)+\\&\begin{bmatrix}-\hat{F}_{2,c}(z^{-1})\\ \tilde{Q}_2(z^{-1})\end{bmatrix}e_2(k)-\\&\begin{bmatrix}\Gamma_{24}+\hat{A}_2\hat{M}_{2c}+z^{-1}\hat{F}_{2c}\\ \Gamma_{28}+\tilde{B}_2\hat{M}_{2c}-z^{-1}\tilde{Q}_2\end{bmatrix}\hat{v}[X(k)],\end{aligned}\tag{40}$$

where

$$\hat{A}_i(z^{-1})=A_i(z^{-1},k),\ \hat{\hat{A}}(z^{-1})=A_i(z^{-1},k-1),$$
$$\hat{B}_i(z^{-1})=B_i(z^{-1},k),\ \hat{\hat{B}}'_i(z^{-1})=B'(z^{-1},k-1),$$
$$B'(z^{-1})=z^{-1}B(z^{-1})=B(z^{-1},k-1),$$

$\tilde{B}_i(z^{-1})$ and $\tilde{Q}_i(z^{-1})$ are determined by the following equations:

$$\begin{aligned}&\tilde{B}_i(z^{-1})[\hat{H}_{ic}(z^{-1})+z^{-1}\hat{\hat{\bar{M}}}_{i,c}(z^{-1})]=\\&\tilde{Q}_i(z^{-1})\hat{\hat{B}}'_i(z^{-1}),\end{aligned}\tag{41}$$

$$\det\tilde{B}_i(z^{-1})=\det\bar{\tilde{B}}'_i(z^{-1}),\tag{42}$$

where $i=1,2$ denote linear and nonlinear, respectively.

$$\begin{aligned}
T(z^{-1})=&\,\hat{F}_{ic}(z^{-1})\hat{\hat{B}}'_i(z^{-1})+\hat{A}_i(z^{-1})[\hat{H}_{ic}(z^{-1})+\\&z^{-1}\hat{\hat{\bar{M}}}_{ic}(z^{-1})],\\
\hat{T}(z^{-1})=&\,\tilde{Q}_i(z^{-1})\hat{\hat{A}}_i(z^{-1})+\tilde{B}_i(z^{-1})\hat{F}_{1c}(z^{-1}),\\
\Gamma_{11}=&\,[\hat{F}_{1c}\cdot\hat{\hat{B}}'_1-\hat{F}_{1c}\hat{\hat{B}}'_1]+[\hat{A}_1\cdot(\hat{H}_{1c}+z^{-1}\hat{\hat{\bar{M}}}_{1c})-\\&\hat{A}_1(\hat{H}_{1c}+z^{-1}\hat{\hat{\bar{M}}}_{1c})],\\
\Gamma_{12}=&\,[\hat{A}_1\cdot\hat{F}_{1c}-\hat{A}_1\hat{F}_{1c}]-[\hat{F}_{1,c}\cdot\hat{\hat{A}}_1-\hat{F}_{1,c}\hat{\hat{A}}_1],\\
\Gamma_{13}=&\,[\hat{A}_1(z^{-1})\cdot\hat{P}_{1c}(z^{-1})-\hat{A}_1(z^{-1})\hat{P}_{1c}(z^{-1})],\\
\Gamma_{14}=&\,\tilde{B}_1\cdot(\hat{H}_{1c}-z^{-1}\hat{\hat{\bar{M}}}_{1c})-\tilde{B}_1(\hat{H}_{1c}-z^{-1}\hat{\hat{\bar{M}}}_{1c})-\\&[\tilde{Q}_1\cdot\hat{\hat{B}}'_1-\tilde{Q}_1\hat{\hat{B}}'_1],\\
\Gamma_{15}=&\,[\tilde{B}_1\cdot\hat{F}_{1c}-\tilde{B}_1\hat{F}_{1c}]+[\tilde{Q}_1\cdot\hat{\hat{A}}_1-\tilde{Q}_1\hat{\hat{A}}_1],\\
\Gamma_{16}=&\,[\tilde{B}_1(z^{-1})\cdot\hat{P}_{1c}(z^{-1})-\tilde{B}_1(z^{-1})\hat{P}_{1c}(z^{-1})],\\
\Gamma_{21}=&\,[\hat{F}_{2c}\cdot\hat{\hat{B}}'_2-\hat{F}_{2c}\hat{\hat{B}}'_2]+[\hat{A}_2\cdot(\hat{H}_{2c}+z^{-1}\hat{\hat{\bar{M}}}_{2c})-\\&\hat{A}_2(\hat{H}_{2c}+z^{-1}\hat{\hat{\bar{M}}}_{2c})],\\
\Gamma_{22}=&\,[\hat{A}_2\cdot\hat{F}_{2c}-\hat{A}_2\hat{F}_{2c}]-[\hat{F}_{2c}\cdot\hat{\hat{A}}_2-\hat{F}_{2c}\hat{\hat{A}}_2],\\
\Gamma_{23}=&\,[\hat{A}_2(z^{-1})\cdot\hat{P}_{2c}(z^{-1})-\hat{A}_2(z^{-1})\hat{P}_{2c}(z^{-1})],\\
\Gamma_{24}=&\,[\hat{A}_2\cdot\hat{M}_{2c}-\hat{A}_2\hat{M}_{2c}],\\
\Gamma_{25}=&\,\tilde{B}_2\cdot(\hat{H}_{2c}+z^{-1}\hat{\hat{\bar{M}}}_{2c})-\tilde{B}_2(\hat{H}_{2c}+z^{-1}\hat{\hat{\bar{M}}}_{2c})-\\&[\tilde{Q}_2\cdot\hat{\hat{B}}'_2-\tilde{Q}_2\hat{\hat{B}}'_2],\\
\Gamma_{26}=&\,[\tilde{B}_2\cdot\hat{F}_{2c}-\tilde{B}_2\cdot\hat{F}_{2c}]+[\tilde{Q}_2\cdot\hat{\hat{A}}_2-\tilde{Q}_2\hat{\hat{A}}_2],\\
\Gamma_{27}=&\,\tilde{B}_2(z^{-1})\cdot\hat{P}_{2c}(z^{-1})-\tilde{B}_2(z^{-1})\hat{P}_{2c}(z^{-1}),\\
\Gamma_{28}=&\,[\tilde{B}_2\cdot\hat{M}_{2c}-\tilde{B}_2\hat{M}_{2c}],
\end{aligned}$$

where the symbols $\hat{*}$ and $\hat{*}'$ represents the estimation of polynomial matrixes at instant k and $k-1$, respectively. The meaning of time-varying operators "$\cdot$" can be seen from [25].

Lemma 3 For n-order time-invariant nonlinear system

$$x(k+1)=\bar{A}x(k)+\bar{B}u(k)+\bar{f}(x(k),u(k)),$$
$$x(0)=x_0,\ y(k)=\bar{C}x(k)+\bar{h}(x(k)),$$

where $x(k),u(k)$ and $y(k)$ are $n\times1$ state vector, $p\times1$ input vector and $m\times1$ output vector, respectively. Assume that $\bar{A}$ is asymptotically stable and $\|\bar{h}(x(k))\|\leqslant\varepsilon_3\|x(k)\|+\varepsilon_4$, $\|\bar{f}(x(k),u(k))\|\leqslant\varepsilon_5\|x(k)\|+\varepsilon_6\|u(k)\|+\varepsilon_7$, where $\varepsilon_i(i=3,\cdots,7)$ is a known positive constants then, if ε_5 satisfies the condition that $0<\varepsilon_5\leqslant1-\bar{K}\lambda$, where $0<\lambda<1$, $0<\bar{K}<\infty$ and satisfies $\|\bar{A}^j\|\leqslant\bar{K}\cdot\lambda^j$, $j=1,\cdots,m$; Then, there exist constants C_1 and

C_2 which are independent of k such that $\|y(k)\| \leqslant C_1 + C_2 \max\limits_{1\leqslant\tau\leqslant k} \|u(\tau)\|$, for $1 \leqslant k \leqslant \infty$, $0 < C_1 < \infty$, $0 < C_2 < \infty$. $i = 1, \cdots, m$.

Theorem 1 Suppose that system (1) satisfies the following assumptions:

i) The linear parameters Θ of the system lies in a compact region Ω with $\det B_0 \neq 0$, $B(1) \neq 0$;

ii) The order n of system is known;

iii) The nonlinearity $v[\cdot]$ satisfies Condition 1.

The weighting constants λ_j, q_j are chosen offline to satisfy Eq. (15). The weighting polynomial matrixes $R_j, K_j(z^{-1})$ and $S_j(z^{-1})$ are chosen online to satisfy Eqs. (34)−(36).

Then when the adaptive decoupling control algorithm (22)−(33) and the switching mechanism are applied to the system (1), the input and output signals in the closed-loop switched system are uniformly bounded(BIBO stability). Furthermore, the nonlinear generalized predictive decoupling controller based on ANFIS can make the generalized tracking error of the closed-loop switched system satisfy

$$\lim_{k\to\infty} \|\bar{e}(k)\| = \lim_{k\to\infty} \|\hat{T}(z^{-1})y(k) - \tilde{B}(z^{-1})\hat{P}_{\mathrm{c}}(z^{-1})w(k+N)\| \leqslant \varepsilon < \infty,$$

where ε is an arbitrarily small positive number.

Proof Firstly, introducing $\tilde{\tilde{A}}_i(z^{-1})$ and $\tilde{\tilde{B}}_i(z^{-1})$ satisfies

$$\hat{A}_i(z^{-1})\tilde{\tilde{B}}_i(z^{-1}) = \tilde{\tilde{B}}'_i(z^{-1})\tilde{\tilde{A}}_i(z^{-1}),\ i = 1, 2, \quad (43)$$

$$\det\tilde{\tilde{B}}_i(z^{-1}) = \det\hat{\tilde{B}}'_i(z^{-1}). \quad (44)$$

From Eqs.(42) and (44), it can be obtained that

$$\det\tilde{\tilde{B}}_i(z^{-1}) = \det\tilde{B}_i(z^{-1}) = \hat{\tilde{B}}'_i(z^{-1}),$$

and combining the Eqs. (41) and (43), it can obtained that

$$\begin{aligned}
&\det\hat{T}(z^{-1}) = \\
&\det[\tilde{Q}_i(z^{-1}\hat{\tilde{A}}_i(z^{-1})) + \tilde{B}_i(z^{-1})\hat{F}_{ic}(z^{-1})] = \\
&\det\{\tilde{B}_i(z^{-1})[\hat{H}_{i,\mathrm{c}}(z^{-1}) + z^{-1}\hat{\tilde{\tilde{M}}}_{i,\mathrm{c}}(z^{-1})]\cdot \\
&(\hat{\tilde{B}}'_i(z^{-1}))^{-1}\hat{\tilde{A}}_i(z^{-1}) + \tilde{B}_i(z^{-1})\hat{F}_{i,\mathrm{c}}(z^{-1})\} = \\
&\det\{\tilde{B}_i(z^{-1})[\hat{F}_{i,\mathrm{c}}(z^{-1})\tilde{\tilde{B}}_i(z^{-1}) + [\hat{H}_{i,\mathrm{c}}(z^{-1}) + \\
&z^{-1}\hat{\tilde{\tilde{M}}}_{i,\mathrm{c}}(z^{-1})](\hat{\tilde{B}}'_i(z^{-1}))^{-1}\hat{\tilde{A}}_i(z^{-1})\tilde{\tilde{B}}_i(z^{-1})]\cdot \\
&(\tilde{\tilde{B}}_i(z^{-1}))^{-1}\} = \\
&\det\{\hat{F}_{i,\mathrm{c}}(z^{-1})\tilde{\tilde{B}}_i(z^{-1}) + [\hat{H}_{i,\mathrm{c}}(z^{-1}) + \\
&z^{-1}\hat{\tilde{\tilde{M}}}_{i,\mathrm{c}}(z^{-1})]\tilde{\tilde{A}}_i(z^{-1})\} = \\
&\det\{[\hat{F}_{i,\mathrm{c}}(z^{-1})\hat{A}_i^{-1}(z^{-1})\hat{\tilde{B}}'_i(z^{-1}) + \\
&[\hat{H}_{i,\mathrm{c}}(z^{-1}) + z^{-1}\hat{\tilde{\tilde{M}}}_{i,\mathrm{c}}(z^{-1})]]\tilde{\tilde{A}}_i(z^{-1})\} = \\
&\det\{\hat{F}_{i,\mathrm{c}}(z^{-1})\hat{\tilde{B}}'_i(z^{-1}) + \hat{A}_i(z^{-1})\times \\
&[\hat{H}_{i,\mathrm{c}}(z^{-1}) + z^{-1}\hat{\tilde{\tilde{M}}}_{i,\mathrm{c}}(z^{-1})]\} = \\
&\det T(z^{-1}).
\end{aligned} \quad (45)$$

The rest of proof is similar to the case of a single variable [17] and thus omitted here due to space limitations.

6 Simulation examples

In order to illustrate the effectiveness of control methods proposed in this paper, simulations have been carried out.

Example 1 Considering the following discrete-time nonlinear systems with unstable zero-dynamics, which is originally adopted from the paper [14].

$$\begin{aligned}
&y_1(k+1) = \\
&1.1y_1(k) + 0.2u_1(k) + u_2(k) + \\
&u_1(k-1) + 1.5y_1(k)\cos(u_1(k) + y_2(k)) + d_1(k), \\
&y_2(k+1) = \\
&0.2y_2(k) + 0.25u_1(k) + 0.2u_2(k) + \\
&u_2(k-1) + 1.2y_2(k)\cos(u_2(k) + y_2(k)) + d_2(k),
\end{aligned}$$

where $d_1(k)$ and $d_2(k)$ are unknown but bounded random disturbance with their upper bounds being given by 0.01 and 0.015, respectively.

The origin is the equilibrium point of system. The parameters matrixes of the system are

$$A(z^{-1}) = \begin{bmatrix} 1 - 1.1z^{-1} & 0 \\ 0 & 1 - 0.2z^{-1} \end{bmatrix},$$

$$B(z^{-1}) = \begin{bmatrix} 0.2 + z^{-1} & 1 \\ 0.25 & 0.2 + z^{-1} \end{bmatrix}.$$

Because $\det B(z^{-1}) = -0.21 + 0.4z^{-1} + z^{-2}$, its eigenvalues are 3.3333 and -1.4286. It can be seen that the zero dynamics of the system is unstable. And the higher order nonlinear terms $1.5y_1(k)\cos(u_1(k) + y_2(k),\ 1.2y_2(k)\cos(u_2(k) + y_2(k)))$ satisfies Condition 1 with $\varepsilon_1 = 0.001, \varepsilon_2 = 0.1$.

The objective of the control is to make the output $y_1(k)$ and $y_2(k)$ asymptotically track the specified bounded signal $w_1(k) = 0.5\,\mathrm{sgn}[\sin(\pi k/50)]$ and $w_2(k) = 0.3$, respectively.

Offline choosing the predictive horizon $N = 1$, the control horizon $N_u = 2$, weighting matrixes $q_j = 0.3I$, $\lambda_j = 0.3I$, $j = 1, \cdots, 3$, then the closed-loop polynomial equation is

$$z^{-1}F_{\rm c}(z^{-1})B(z^{-1}) + A(z^{-1})[H_{\rm c}(z^{-1}) + z^{-1}\bar{\bar{M}}_{\rm c}(z^{-1})] = 1.2671 - 1.2851z^{-1} + 0.2860z^{-3} - 0.0484z^{-4}.$$

Its eigenvalues are $-0.4489, 0.8350, 0.5054$ and 0.2016 which all lie inside the unit circle.

The membership function of ANFIS is set as Gaussian function, whose number is 3. The parameters in switching function are $c = 0.2$, $T = 3$. The effectiveness of the simulation can be seen from Figs. 2$-$3.

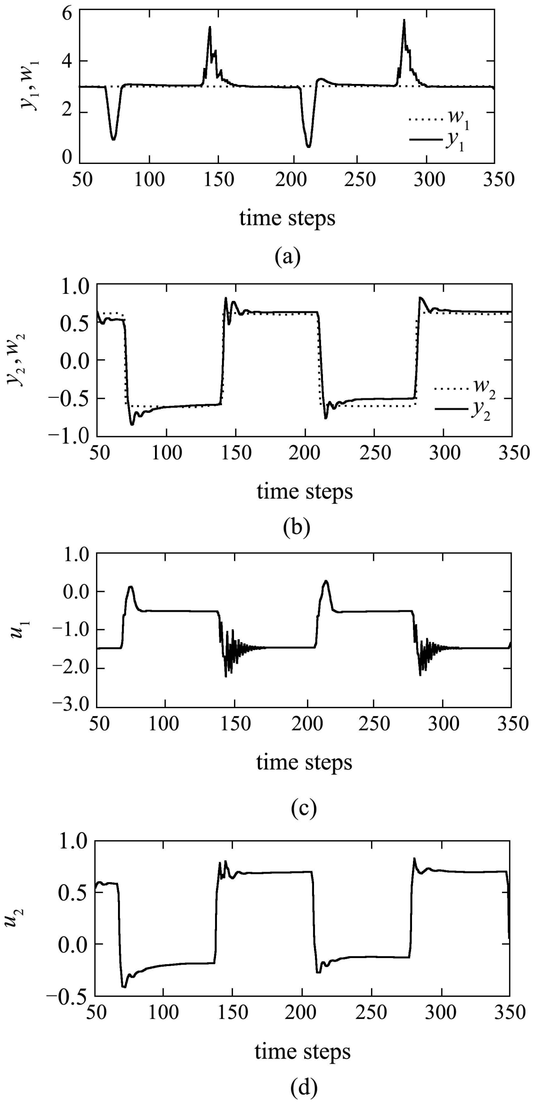

Fig. 2 Performance of the closed-loop switching system

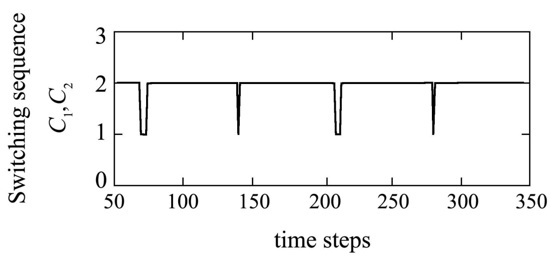

Fig. 3 Switching sequence (1: linear; 2: nonlinear)

Figure 2 shows the performance of the closed-loop switching systems, where Fig.2(a) is the output $y_1(k)$ and reference trajectory $w_1(k)$, Fig.2(b) is output $y_2(k)$ and reference trajectory $w_2(k)$. Fig.2(c) and Fig.2(d) are the input $u_1(k)$ and $u_2(k)$, respectively. From Fig.2, it can be seen that the closed-loop is stable and the good tracking performance has been achieved.

Figure 3 shows the switching sequence of the system, where $k = 1$ denotes the use of linear generalized predictive decoupling controller and $k = 2$ stands for the use of nonlinear generalized predictive decoupling controller based on ANFIS. As it can be seen from Fig.3, the nonlinear decoupling controller has worked very well during most of the time and it degrades the performance sometimes. In order to ensure the stability of the system, the linear decoupling controller had to take over until the nonlinear decoupling controller is recovered.

7 Conclusion

In this paper, a nonlinear adaptive generalized predictive decoupling control method based on unmodeled dynamic compensation is proposed for a class of discrete-time MIMO nonlinear dynamic systems with NARMA form, where the ANFIS is combined with the multiple models adaptive control method. The proposed method relax the global boundedness assumption of the higher-order nonlinear term and thus enhance the practicability of the method. The stability and the convergence of the closed-loop system are analyzed and simulation results have demonstrated the effectiveness of the proposed approach.

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